Statistics 431: Statistical Inference
Fall 2006

Problem Set 3 — Due 9 Oct 2006

Devore Chapter 8: #46, 50, 52, 59, 62, 69, 70, 76.
Note: In question 59, assume normality.

Additionally:

A. What is the p-value for the situation in problem set 2, problem B? How is this p-value related to your conclusion in part (b) of that problem? How is the confidence interval in part (c) of that problem related to the conclusion of the test in part (b)?

B. How is the CI in problem set 1, problem B related to the result of the hypothesis test in problem set 2, problem A(a)?

C. A blood test intended to identify patients with cardiac disease gave positive results on 50 out of 60 known cardiac patients, but also on 16 out of 200 known healthy (non-cardiac) patients.

(a) Find a 95% CI for the sensitivity of the test, which is defined here as the probability that a cardiac patient is correctly identified. Would a test of $H_0 : p = 0.8$ versus $H_A : p \neq 0.8$ accept or reject at $\alpha = 0.05$?

(b) Find a 95% CI for the specificity of the test, which is defined here as the probability that a healthy patient is correctly identified. Would a test of $H_0 : p = 0.8$ versus $H_A : p \neq 0.8$ accept or reject at $\alpha = 0.05$? (Note: It is usually desirable to have sensitivity $\geq$ specificity. Why? But that appears not to be the case for this blood test.)

D. Many additional cardiac patients were then given the blood test in problem C, and the sensitivity of that test was determined to be almost exactly 80%. A new testing method has been derived that supplements the blood test with information based on electrocardiogram measurements. This new method was tested on 55 cardiac patients and gave correct results for 51 of them. Is this convincing evidence that the new method has a higher sensitivity than the old method?

(a) Set up the hypotheses to validate the theory that the sensitivity of the new method is better than that of the old. (It is assumed that the sensitivity cannot be worse.)

(b) Give the p-value for this test corresponding to the observed data. Does a level $\alpha = 0.05$ test reject or accept? Did you use the large-sample (normal) approximation or the small-sample (binomial) procedure? Why? Would it make a difference in your result?