Statistics 430, Spring 2007.

Full credit will only be given for complete, clear and accurate answers. Show all your work. You may find it helpful to recall the following. Conditional probability is defined by the formula

$$P(AB) = P(A|B)P(B).$$

The variance of a random variable X is given by

$$Var(X) = E(X - EX)^2 = EX^2 - (EX)^2$$

The density function of an exponential distribution is given by

$$f(x) = \lambda e^{-\lambda x}$$

for $x \geq 0$ otherwise it is zero. The mean and standard deviation of the exponential distribution is $\frac{1}{\lambda}$.

1. In an urn there are five balls, numbered 1,2,3,4 and 5. Balls are picked out at random and then replaced so that on each pick each number is equally likely to be chosen. Let X_i be the number chosen on the ith pick.
   (a) Compute $E(X_1 - 3)^4$
   (b) Compute $Var(\bar{X})$ where $\bar{X} = \frac{1}{625}\sum_{i=1}^{625} X_i$.
2. A die is thrown many times.
   (a) What is the expected value of the number of throws until either a 2 or a 3 have been observed?
   (b) What is the expected value of the number of throws until both a 2 and a 3 have been observed?
3. A particle counter records two types of particles The Type A particles arrive at an average rate of 2 per minute according to a Poisson process. The Type B particles arrive independently of the type A particles at a rate of 4 per minute also according to a Poisson process.
   (a) Find the probability that exactly 4 Type A particles arrive and exactly 8 Type B particles arrive in the first 2.5 minutes.
   (b) Find the chance that the first arrival is within the first 3 minutes given that there are exactly two arrivals within the first 4 minutes.
   (c) The first particle to arrive is of Type A given that only one particle arrived in the first t minutes.
4. Suppose that we have two decks of 52 cards. Both decks are shuffled and the cards are matched against each other one at a time. A match is said to occur at position i if the ith card drawn from each deck is the same. Let N be the total number of matches.

(a) Use the Poisson approximation to approximate the chance that $N \leq 2$.

5. For a group of n people find the expected number of days for which at least 2 of these people share as a birthday. You should assume that all arrangements of birthdays are equally likely and that there are 365 days in the year.

6. Let X, Y, Z be independent random variables each with cdf $F(x) = x^3$ for $0 \leq x \leq 1$. Find

   (a) $E(XYZ)$

   (b) $E(max(X, Y, Z))$

   (c) $P(min(X, Y, Z) > 0.5)$