

Section 3.2

6. Let A_i denote the event that i th card is spade. Then $X = \sum_{i=1}^7 1_{A_i}$, where 1_{A_i} is the indicator function of A_i . $EX = E \sum_{i=1}^7 1_{A_i} = \sum_{i=1}^7 E1_{A_i} = \sum_{i=1}^7 P(A_i) = 7 * \frac{13}{52} = \frac{7}{4}$.

8. $E[(X+Y)^2] = E[X^2 + 2XY + Y^2] = EX^2 + 2EXY + EY^2 = 3 + 2*2 + 4 = 11$.

Section 3.3

2.

Y	0	1	2	3
Y^2	0	1	4	9
Y^4	0	1	16	81
Pr	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$EY^2 = 1 * \frac{3}{8} + 4 * \frac{3}{8} + 9 * \frac{1}{8} = 3$, $EY^4 = 1 * \frac{3}{8} + 16 * \frac{3}{8} + 81 * \frac{1}{8} = \frac{132}{8}$, so $VarY^2 = EY^4 - (EY^2)^2 = 7.5$.

3.

(a) $E(2X + 3Y) = 2EX + 3EY = 2 * 1 + 3 * 1 = 5$

(b) $Var(2X + 3Y) = 2^2VarX + 3^2VarY + 2Cov(2X, 3Y) = 4*2 + 9*2 + 0 = 26$

(c) $EXYZ = EXEYEZ = 1$

(d)

$$\begin{aligned} Var(XYZ) &= EX^2Y^2Z^2 - (EXYZ)^2 \\ &= EX^2EY^2EZ^2 - (EXEYEZ)^2 \\ &= (VarX + (EX)^2)(VarY + (EY)^2)(VarZ + (EZ)^2) - (EXEYEZ)^2 \\ &= (2 + 1)^3 - 1 \\ &= 26 \end{aligned}$$

4.

$$\begin{aligned} Var(X_1X_2) &= EX_1^2X_2^2 - (EX_1X_2)^2 \\ &= EX_1^2EX_2^2 - (EX_1EX_2)^2 \\ &= (VarX_1 + (EX_1)^2)(VarX_2 + (EX_2)^2) - (EX_1EX_2)^2 \\ &= (\sigma_1^2 + \mu_1^2)(\sigma_2^2 + \mu_2^2) - \mu_1^2\mu_2^2 \\ &= \sigma_1^2\sigma_2^2 + \mu_1^2\sigma_2^2 + \sigma_1^2\mu_2^2 \end{aligned}$$

5.

$$\begin{aligned} E[(X - a)^2] &= E(X - \mu + \mu - a)^2 \\ &= E[(X - \mu)^2 + (\mu - a)^2 + 2(X - \mu)(\mu - a)] \\ &= \sigma^2 + (\mu - a)^2 + 2(\mu - a)(EX - \mu) \\ &= \sigma^2 + (\mu - a)^2 \end{aligned}$$

9. Let Z denote the number of Republican votes at the second election, X denote the number of Republican votes from Republican at the first election, and Y denote the number of Republican votes from Democrat at the first election. X and Y are independent, and $Z = X + Y$. And we have $X \sim \text{Bin}(r, 1 - p_1)$, $Y \sim \text{Bin}(n - r, p_2)$. $EX = r(1 - p_1)$, $EY = (n - r)p_2$, $\text{Var}X = r(1 - p_1)p_1$, and $\text{Var}Y = (n - r)p_2(1 - p_2)$. Hence $EZ = r(1 - p_1) + (n - r)p_2$, and $\text{Var}Z = r(1 - p_1)p_1 + (n - r)p_2(1 - p_2)$.