

The following problems are from J. L. Devore, *Probability and Statistics for Engineering and the Sciences*, 7th ed.

Problems: 13.24, 13.56, 13.58, 13.60, 13.62, 13.64, 13.68, 13.72, 13.80.

All of the following problems refer to the logistic distribution function

$$\Lambda(x) = \frac{e^x}{1 + e^x}.$$

When solving the problems you might have to look up the lecture notes.

1. Check that the logistic distribution is symmetric, i.e., $1 - \Lambda(x) = \Lambda(-x)$.
2. (a) If $\mathbf{P}(Y_i = 1|X) = \Lambda(X_i\beta)$, show that $\text{logit}\mathbf{P}(Y_i = 1|X) = X_i\beta$.
(b) If $\text{logit}\mathbf{P}(Y_i = 1|X) = X_i\beta$, show that $\mathbf{P}(Y_i = 1|X) = \Lambda(X_i\beta)$.
3. Show that for the logistic regression model the log likelihood function is given by

$$l_n(\beta) = - \left(\sum_{i=1}^n \ln [1 + e^{X_i\beta}] \right) + \left(\sum_{i=1}^n X_i Y_i \right) \beta.$$