

## Practice Problems for Midterm 1

*Problem 1.* Suppose that each child born to a couple is equally likely to be a boy or a girl independent of the sex distribution of the other children in the family. For a couple having 5 children, compute the probabilities of the following events:

- (a). All children are of the same sex;
- (b). The 3 eldest are boys and the others girls;
- (c). Exactly 3 are boys;
- (d). The 2 eldest are girls;
- (e). There are at least 1 girl.

*Problem 2.* In a certain community, 8% of all adults over 50 have diabetes. If a health service in this community correctly diagnoses 95% of all persons with diabetes as having the disease and incorrectly diagnoses 2% of all persons without diabetes as having the disease, find the probabilities that

- (a). the community health service will diagnose a randomly selected adult over 50 as having diabetes;
- (b). a person over 50 diagnosed by the health service as having diabetes actually has the disease.

*Problem 3.* An urn contains 10 red balls and 5 blue balls. Five balls are chosen at random with replacement from this urn.

- (a). What is the probability that all five balls drawn are red?
- (b). What is the probability that among those five there are at least one red ball and also at least one blue ball?

*Problem 4.* Suppose that the 10 red balls are labeled as  $R_1, R_2, \dots, R_{10}$  and they are randomly arranged in a single line. What is the probability that  $R_1, R_2$  and  $R_3$  are in adjacent positions?

*Problem 5.* Two fair dice are rolled. Let  $X$  and  $Y$  denote, respectively, the maximum and minimum of the two rolls.

- (a). Find the joint pmf of  $X$  and  $Y$ .
- (b). Find the marginal pmf of  $X$ .
- (c). Find the conditional pmf of  $Y$  given that  $X = 3$ .

- (d). Calculate  $E(X + Y)$ .
- (e). Are  $X$  and  $Y$  independent? Why?

*Problem 6.* Suppose  $X$  is binomial with  $n = 23$  and  $p = 0.7$ . Let  $Y = 23 - X$ . (So, if  $X$  is the number of "successes" then  $Y$  is the number of "failures".) Find:

- (a).  $E(X - Y)$ .
- (b).  $E((X - Y)^2)$ .

*Problem 7.*  $A$  and  $B$  alternate rolling a pair of dice, stopping either when  $A$  rolls the sum 9 or when  $B$  rolls the sum 6. Assuming that  $A$  rolls first, find the probability that the final roll is made by  $A$ .

*Problem 8.* When a randomly chosen person is offered a free pizza of any brand they choose, 1 of 10 people choose "Peace-a-Pizza".

- (a). If 3,600 randomly chosen people are made this offer, approximately what is the probability that at least 350 will choose Peace-a-Pizza?
- (b). Peace-a-Pizza would like to create an ad campaign to raise their popularity considerably above the current 10% level. They would like it to be the case that when 3,600 randomly chosen people are made this offer at least 600 would choose their pizza. To what level do they need to raise their popularity in order to be 80% certain that among 3,600 randomly chosen people at least 600 will choose Peace-a-Pizza pizza? [Use suitable normal approximations to answer both parts of this question.]

*Problem 9.* In a certain community, 36% of the families own a dog, 30% of the families own a cat, and 22% of the families that own a dog also own a cat. What is

- (a). the probability that a randomly selected family owns both a dog and a cat?
- (b). the conditional probability that a randomly selected family owns a dog given that it owns a cat?

*Problem 10.* Consider 3 urns. Urn  $A$  contains 2 white and 4 red balls; Urn  $B$  contains 8 white and 4 red balls; and Urn  $C$  contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn  $A$  was white, given that exactly 2 white balls were selected?

*Problem 11.* The intelligence quotient (IQ) test score is normally distributed with mean 100 and standard deviation 16.

- (a). What is the probability that at least two of five randomly selected people have IQ test score higher than 120?
- (b). A person is randomly selected from all people whose IQ are higher than 110. What is the probability that this person has IQ higher than 120?
- (b). One thousand people are randomly selected. Approximately what is the probability that fewer than 90 of them have IQ score higher than 120?