Comments on *The Cauchy-Schwarz Master Class*

1. Page 50, Exercise 3.8. The defect R in equation (3.16) is also the defect in the inequality

$$\sum_{j=1}^{n} \frac{a_j b_j}{a_i + b_j} \quad \leq \quad \left(\sum_{j=1}^{n} \frac{a_j^2}{a_i + b_j} \right)^{1/2} \left(\sum_{j=1}^{n} \frac{b_j^2}{a_i + b_j} \right)^{1/2}.$$

2. Page 65, lines 7 and 8. In the two inequalities, the term B should be B^2.

3. Page 118, Exercise 7.11. Inequality (7.27) is worse than Jensen's inequality applied to the function $\ln f(X)$ of the random variable X that is distributed on $[a, b]$ with density $w(x)$.

4. Page 129, line -14. "apply continuous version" should be "apply the continuous version".

5. Page 158, line 1. The function $f(x) = m^{2\lambda} x^{2\lambda} (m + x)^{-1}$ is missing a minus sign in the exponent of $x^{2\lambda}$.

6. Page 165, line 13. The double sum on the right for I is missing a factor of $1/i$.

7. Page 175, line 2. In formula (11.17) the subscript of a should be n rather than k.

8. Page 205, Exercise 13.5, line -1. The function $-\sum_{k=1}^{n} p_k \ln p_k$ is Schur concave.

9. Page 212, equations (14.11) and (14.12). The denominator $\sin(\pi h \alpha)$ is missing a factor of 2 in its argument. This propagates to a missing 2 in the denominator of $\|h\alpha\|$ and possibly further in the problem.

10. Page 222, Exercise 14.5. Formula (14.36) requires that j and m have greatest common divisor 1, not that m does not divide j.

11. Page 233, line 4. Replace "both" with "all three".

12. Page 236, lines 4 and 5. I was able to use Cauchy's trick of padding a sequence.

13. Page 241, line 2. The inequality should read

$$\left| \frac{1}{z_1} - \frac{1}{z_3} \right| \quad \leq \quad \left| \frac{1}{z_1} - \frac{1}{z_2} \right| + \left| \frac{1}{z_2} - \frac{1}{z_3} \right|.$$

14. It took me "all day" to come up with the following solution:

It is easy to show by convexity or algebra that

$$\frac{1}{1+u} \leq 1 - \frac{u}{2}$$

for $u \in [0,1]$ and

$$\frac{1}{1+u} \leq 1 - \frac{u}{3}$$

for $u \in [0,2]$. Hence,

$$\begin{aligned} & \frac{x^2}{1+y} + \frac{y^2}{1+z} + \frac{z^2}{1+x+y} + x^2(y^2-1)(z^2-1) \\ \leq\ & x^2\Big(1 - \frac{y}{2}\Big) + y^2\Big(1 - \frac{z}{2}\Big) + z^2\Big(1 - \frac{x+y}{3}\Big) + x^2(y^2-1)(z^2-1) \\ =\ & g(x,y,z) \end{aligned}$$

on the stated domain $x, y, z \in [0,1]$. The second derivative test demonstrates that $g(x,y,z)$ is convex in x for y and z fixed. Hence, $g(x,y,z)$ attains its maximum at $x = 0$ or $x = 1$. Now

$$\begin{aligned} g(1,y,z) &= 1 - \frac{y}{2} + y^2\Big(1 - \frac{z}{2}\Big) + z^2\Big(\frac{2}{3} - \frac{y}{3}\Big) + (y^2-1)(z^2-1) \\ g(0,y,z) &= y^2\Big(1 - \frac{z}{2}\Big) + z^2\Big(1 - \frac{y}{3}\Big), \end{aligned}$$

and so

$$\begin{aligned} g(1,y,z) - g(0,y,z) &= 1 - \frac{y}{2} - \frac{z^2}{3} + (y^2-1)(z^2-1) \\ &\geq 0 \end{aligned}$$

since $1 \geq \frac{1}{2}y + \frac{1}{3}z^2$ and the product $(y^2-1)(z^2-1)$ is nonnegative. Finally, $g(1,y,z) \leq 2$, if and only if

$$1 - \frac{y}{2} + y^2\Big(1 - \frac{z}{2}\Big) + z^2\Big(\frac{2}{3} - \frac{y}{3}\Big) + y^2z^2 - y^2 - z^2 + 1 \leq 2,$$

if and only if

$$-\frac{1}{2}y - \frac{1}{2}y^2z - \frac{1}{3}z^2y + y^2z^2 - \frac{1}{3}z^2 \leq 0.$$

The last inequality is true because

$$y^2z^2 \leq \frac{1}{2}y + \frac{1}{2}y^2z.$$

15. Page 252, line - 11. The domain of $f(x)$ should be $-\pi/2 < \theta < \pi/2$.
16. Page 257, line - 2. "Fist" should be "first."
17. Page 283, line 7. Why is it possible to include all points rather than just those in the halfplane S_θ? The limits of integration on the integral also do not make sense.
18. Page 283, line 11. The reference Bledsoe (1970) is missing in the references.