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SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

DISTINCT SUMS OVER SUBSETS

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ABSTRACT. A finite set of integers with distinct subset sums has a precisely bounded Dirichlet series.

Let $1 \leqslant a_1 < a_2 < \cdots < a_n$ be a set of integers for which all of the sums $\sum_{i=1}^{n} \varepsilon_i a_i$, $\varepsilon_i = 0$ or 1, are distinct. It was conjectured by P. Erdös and proved by C. Ryavec that

$$\sum_{i=1}^{n} \frac{1}{a_i} < 2.$$

We will show that for all real $s \geqslant 0$,

$$\sum_{i=1}^{n} \left(\frac{1}{a_i}\right)^s < \frac{1}{1-2^{-s}}.$$

The hypothesis clearly implies for $0 < x < 1$ that

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k > (1+x^{a_1})(1+x^{a_2}) \cdots (1+x^{a_n})$$

and

$$-\log(1-x) > \sum_{i=1}^{n} \log(1+x^{a_i}),$$

as was observed in [1].

The crucial idea here is that the form $|\log x|^\beta \, dx/x$ is changed only by a constant factor under the substitution $y = x^a$, so integrating we have

$$\int_0^1 |\log(1-x)| \, |\log x|^\beta \, \frac{dx}{x}$$
$$> \int_0^1 \log(1+y) |\log y|^\beta \, \frac{dy}{y} \sum_{i=1}^{n} \left(\frac{1}{a_i}\right)^{1+\beta}.$$

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To calculate the first integral we substitute $x = e^{-u}$, expand $\log(1 - e^{-u})$, and integrate term-by-term to obtain $\Gamma(\beta + 1)\zeta(\beta + 2)$, where ζ is the Riemann zeta function. In the same way the second integral is found to be $\Gamma(\beta + 1)\zeta(\beta + 2)(1 - (\frac{1}{2})^{\beta+1})$. These calculations are valid for all $\beta > -1$, so the theorem follows.

BIBLIOGRAPHY

1. S. J. Benkowski and P. Erdős, *On weird and pseudoperfect numbers*, Math. Comp. 28 (1974), 617–623.

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