Discussion of

On the Problem of Evaluating Formal Posterior Distributions

by Morris Eaton

and

Reference Priors Under Partial Invariance

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Evaluating default Bayesian methodology

Model selection	Estimation
Consistency	Admissibility
Mean squared error	Frequentist coverage

- Evaluate formal Bayesian procedures by means of frequentist calculations
- Proper priors result in admissible procedures
- Complete class theorems: admissible are formal Bayes
- Strong admissibility: is it really what we want from a "non-informative" procedure?
- "boundary of admissibility" — Berger and Strawderman (96)

Admissibility of formal Bayes procedures

- Brown (71) — recurrent diffusion on the sample space $\mathcal{X}$
- Eaton (92) — recurrent discrete time Markov chain on the parameter space Θ
- Cast admissibility as a minimization problem using Stein (55) and Blyth (51)

Eaton (92) transition kernel of the MC:

$$K(\theta' \mid \theta) = \int_{\mathcal{X}} \frac{f(x \mid \theta')\pi^N(\theta')}{m(x)} f(x \mid \theta) dx$$

$$\pi^N(\theta) K(\theta' \mid \theta) = \pi^N(\theta') K(\theta \mid \theta')$$

$\therefore \pi^N(\theta)$ is the invariant measure (unique up to a multiplicative constant) for K — but π^N is only σ-finite...

Verifiable conditions

- Brown (71)
  - conditions on $m(x)$
- Eaton (08)
  - $\pi^N(\theta) = \int \pi(\theta \mid \beta)\pi^N(\beta)d\beta$
  - sufficient conditions for recurrence on moments of increments of the chain

Invariance and invariant priors

- Typically simplify calculations
- Known general results?

Semi-invariance

- Semi-invariance $\sim$ partial information
- Sun and Berger (98) "Reference priors with partial information"— for $\pi(\theta_1, \theta_2)$
  - know $\pi(\theta_1)$, want $\pi(\theta_2 \mid \theta_1)$
  - know $\pi(\theta_1 \mid \theta_2)$, want $\pi(\theta_2)$
  - know that *a priori* θ_1 and θ_2 are independent, want joint
- One can argue that since $f(x \mid \theta, \xi)$ has the same invariance structure regardless of θ, $\pi(\xi \mid \theta) = \pi^r(\xi)$
- Once can argue that one wants to take advantage of invariance, so that $\pi(\xi) = \pi^r(\xi)$
- One can even argue for independence...

Applications to testing?

$$H_0 : X \sim \frac{1}{\sigma}\, g_1\left(\frac{x-\mu}{\sigma} \mid \theta_1\right) \text{ vs } H_1 : X \sim \frac{1}{\sigma}\, g_2\left(\frac{x-\mu}{\sigma} \mid \theta_2\right)$$

- reparameterize if necessary to guarantee that the group actions are the same under H_0 and H_1, leaving θ_1 and θ_2 unaffected
- $\pi^R(\theta_1)$ and $\pi^R(\theta_2)$ proper
- $m_1(x \mid \mu, \sigma)$ and $m_2(x \mid \mu, \sigma)$ retain the invariance structure (Dass and Berger, 03)
- $B_{12} = f_1(T(x))/f_2(T(x))$ (Berger, Pericchi and Varshavsky, 98)