nate of

Yes → $\left(\bar{y} - t_{\alpha/2,\,n-1}\frac{s}{\sqrt{n}},\ \bar{y} + t_{\alpha/2,\,n-1}\frac{s}{\sqrt{n}}\right)$

$t_{\alpha/2,\,n-1}$ may be replaced by $z_{\alpha/2}$

Yes → $\left(\bar{y} - t_{\alpha/2,\,n-1}\frac{s}{\sqrt{n}},\ \bar{y} + t_{\alpha/2,\,n-1}\frac{s}{\sqrt{n}}\right)$

s

ean	StDev	SE Mean	90% CI
600	9.182	4.106	(94.846, 112.354)

ean	StDev	SE Mean	95% CI
600	9.182	4.106	(92.200, 115.000)

ean	StDev	SE Mean	99% CI
.600	9.182	4.106	(84.695, 122.505)

le size (N), the sample mean (Mean), the sample standard devia-
ean (SE Mean) and the confidence interval for the requested level

The quantities needed to construct the interval are

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = 103.6$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}} = 9.18$$

$$t_{0.025,4} = 2.776$$

The point estimate of μ is $\bar{y} = 103.6$. The interval estimate of μ is

$$\left[103.6 - 2.776\left(\frac{9.18}{\sqrt{5}}\right),\ 103.6 + 2.776\left(\frac{9.18}{\sqrt{5}}\right)\right] \quad \text{or} \quad (92.2, 115.0)$$

Computer packages such as Excel, MINITAB, and SAS also can be used to construct confidence intervals. Figure 2.21 shows the MINITAB output for requests for 90%, 95%, and 99% confidence intervals using the data from Example 2.6. There is more information on such procedures in the Using the Computer section at the end of this chapter. ■

EXERCISES

22. A local department store wants to determine the average age of the adults in its existing marketing area to help target its advertising. A random sample of 400 adults is selected. The sample mean age is found to be 35 years with a sample standard deviation of 5 years. Construct a 95% confidence interval estimate of the population average age of the adults in the area.

23. The management of a manufacturing plant is studying the number of times employees in a large population of workers are absent. A random sample of 25 employees is chosen, and the average number of annual absences per employee in the sample is found to be six. The sample standard deviation is 0.6. Assuming the population of absences is normally distributed, construct a 99% confidence interval estimate of the population average number of absences.

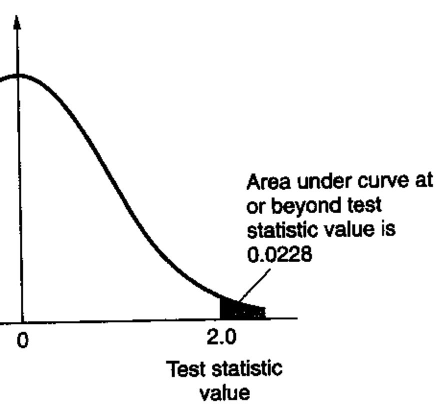
24. A quality control inspector is concerned with the average amount of weight that can be held by a type of steel beam. A random sample of five weight added before the beams begin to show stress (in thousands of pounds):

9, 11, 10, 10, 8

Assuming that the population of weights is normally distributed, construct a 95% confidence interval estimate of the population average weight that can be held.

25. Refer to the data in Exercise 1. Highway mileages of 147 cars are in a file named CARS2. Assume these cars represent a random sample of all new cars produced in 2003. Find a 95% confidence interval estimate for the population mean miles per gallon.

26. The July 1, 2002, one-year returns for a random sample of 83 mutual funds are available in a data file named ONERET2. Find a 95% confidence interval estimate for the population mean rate of return.
(*Source*: The data are used by permission from the September issue of *Kiplinger's Personal Finance*. Copyright © 2002 The



of significance, our decision is to reject the null hypothesis .0228 is less than 0.05.

h approach to hypothesis testing is chosen (unstandardized istic or p value), the decision made is identical for a given

are computed differs depending on whether the test is an ed, or two-tailed test. In all three cases, the first step is to d test statistic. For an upper-tailed test (as in the previous il- probability to the right of the standardized test statistic. For pute the probability to the left of the standardized test statis- t, compute the probability in the tail area closest to the stan- and then multiply this area by 2. By computing p values in ng decision rule can always be used:

e $< \alpha$

p value $\geq \alpha$

MINITAB output for testing the hypothesis discussed in t shows $\bar{y} = 103.600$, $s = 9.182$, $s/\sqrt{n} = 4.106$, $t = -1.56$,)97. Using the p value decision rule, note that the null ted for levels of significance of 1% or 5%, but is rejected at

10%. Reporting the p value is somewhat more informative in a case such as this than simply reporting a decision to reject or fail to reject the null hypothesis.[3] ■

EXERCISES

27. If a null hypothesis is rejected at the 5% level of significance, what decision would have been made at the 10% level? Why?

28. To investigate an alleged unfair trade practice, the Federal Trade Commission (FTC) takes a random sample of sixteen "5-ounce" candy bars from a large shipment. The mean of the sample weights is 4.85 ounces and the sample standard deviation is 0.1 ounce. Test the hypotheses

$$H_0: \mu \geq 5$$
$$H_a: \mu < 5$$

at the 5% level of significance. Assume the population of candy bar weights is approximately normally distributed. Based on the results of the test, does the FTC have grounds to proceed against the manufacturer for the unfair practice of short-weight selling? State the decision rule, the test statistic, and your decision.

29. A quality inspector is interested in the time spent replacing defective parts in one of the company's products. The average time spent should be at most 20 minutes (min) per day according to company standards. The following hypotheses are set up to examine whether the standards are being met:

$$H_0: \mu \leq 20$$
$$H_a: \mu > 20$$

where μ represents the population average time spent replacing defective parts. To conduct the test, a random sample of 16 employees is chosen. The average time spent replacing defective parts for the sample was 20.5 min, with a sample standard deviation of 4 min. Perform the test at a 5% level of significance. Assume the population of service times is approximately normally distributed. State the decision rule, the test statistic, and your decision. Are company standards being met?

30. Refer to the data in Exercise 1. Highway mileages of 147 cars are in a file named CARS2. Assume these cars represent a random sample of all new cars produced in 2003. The corporate average fuel economy (CAFE) standards set by the government require that average fuel economy for cars be more than 27.5 miles per gallon. To examine whether the CAFE standard is being met, the following hypotheses are tested:

$$H_0: \mu \leq 27.5$$
$$H_a: \mu > 27.5$$

where μ is the population average gas mileage for all 2003 cars. Use a 5% level of significance. State the decision rule, the test statistic, and your decision. What is your conclusion regarding the average mileage of 2003 cars?

31. Refer to the data in Exercise 26. The July 1, 2002, one-year returns for a random sample of 83 mutual funds are available in a file named ONERET2. The return for the S&P 500 stock index for the same one-year time period was −18.0%. Test to see if there is evidence that the average one-year return for the population of funds is more than the return for the S&P 500 stock index. Use a 5% level of significance. State the hypotheses to be tested, the decision rule, the test statistic, and your decision. What is your conclusion regarding the average one-year return for mutual funds?

Construct a 95% confidence interval estimate of the difference between the mean GMAT scores for the two groups.

33. Two suppliers are being considered by a manufacturer. Independent random samples of ten parts from shipments from each supplier are selected, and the lifetime in hours for each part is determined for each sample. Use the following information to construct a 98% confidence interval estimate of the difference in the population average lifetimes. Assume that the population variances are equal and the populations are normally distributed.

	Supplier 1	Supplier 2
Sample size	10.0	10.0
Sample mean	15.0	11.0
Sample standard deviation	1.5	1.0

34. To help validate a new employee-rating form, a company administers it to independent random samples of employees in two different divisions. The following information is obtained from the scores on the forms:

	Division 1	Division 2
Sample size	15.0	15.0
Sample mean	82.0	78.0
Sample standard deviation	3.0	2.5

Use the information to construct a 95% confidence interval estimate of the difference in mean scores between the two divisions. Assume that the population variances are equal and the populations are normally distributed.

35. The one-year returns for a random sample of 51 load mutual funds and 32 no-load funds were obtained. The returns are in a file named RETURNS2. Construct a 95% confidence interval estimate of the difference between the population mean returns.

These data are arranged in the file in two columns. The first column contains the returns for the funds and the second column indicates to which sample (load = 1, no-load = 0) each value in column 1 belongs (in the Excel spreadsheet, the returns are in two separate columns).

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36. The file named PRIVATE2 contains the graduation rates for 195 schools and a variable coded 1 for private schools and 0 for public schools. Construct a 99% confidence interval estimate for the difference between population average graduation rates for public and private schools.

These data are arranged in the file in two columns. The first column contains the graduation rates and the second column contains the private school variable (in the Excel spreadsheet, the graduation rates are in two separate columns for private and public schools).

(*Source*: Used by permission from the November and December 2003 issues of *Kiplinger's Personal Finance*. Copyright © 2003 The Kiplinger Washington Editors, Inc. Visit our website at www.kiplingers.com for further information).

2.9 HYPOTHESIS TESTS ABOUT THE DIFFERENCE BETWEEN TWO POPULATION MEANS

We may be interested in testing hypotheses about the difference between two population means rather than estimating that difference. The most common hypotheses tested in comparing two populations are

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

The null hypothesis states that the means of the two populations are equal, whereas

restated in terms of the difference betweeń two

$$H_0: \mu_1 - \mu_2$$
$$H_a: \mu_1 - \mu_2$$

The decision rule for the test is

Reject H_0 if $t > t_{\alpha/2,df}$ or $t < -t_{\alpha/2,df}$

Do not reject H_0 if $-t_{\alpha/2,df} \leq t \leq t_{\alpha/2,df}$

The construction of the test statistic, t, depends can be assumed equal. If $\sigma_1^2 = \sigma_2^2$, then

$$t = \frac{\bar{y}_1 - \bar{y}}{\sqrt{s_p^2\left(\frac{1}{n_1} + \right.}}$$

and the critical value, $t_{\alpha/2,df}$, is chosen with df = The pooled estimate of the population variance dard deviation of the sampling distribution.

If $\sigma_1^2 \neq \sigma_2^2$, then

$$t = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s}{n}\right.}}$$

and the approximate critical value is chosen with

$$\Delta = \frac{(s_1^2/n_1 + s_2^2/}{[(s_1^2/n_1)^2/(n_1 - 1)] + [(s}$$

degrees of freedom.

The justification for using two different stan statistics is the same as that for constructing the c section.

Figure 2.28 shows the three possible hypoth be used, and the decision rules for the case whe similar information for $\sigma_1^2 \neq \sigma_2^2$.

EXAMPLE 2.11 Consider again the 5-year return data for the examined in Example 2.10 of Section 2.8 (●).

Let μ_L represent the population mean 5-year resent the population mean 5-year return for no-lo

$$H_0: \mu_N - \mu_L =$$

Based on the test results, the null hypothesis should not be rejected. The t statistic or the p value can be used to reach this decision. If the t statistic is used, the decision rule is

Reject H_0 if $t > 1.96$ or $t < -1.96$

Do not reject H_0 if $-1.96 \leq t \leq 1.96$

There are 56 degrees of freedom for this test, so the z value of 1.96 was used as the critical value. Note that Excel provides the t value for 56 degrees of freedom for a two-tailed test: $t = 2.00$. This value could be used (and is in fact preferred) rather than the z value if it is available. The test statistic value is $t = 0.76$.

If the p value is used, the decision rule is

Reject H_0 if p value < 0.05

Do not reject H_0 if p value ≥ 0.05

where the p value is 0.45. As always, both procedures lead to the same decision.

The statistical decision is do not reject the null hypothesis, so we conclude that there is no difference between the average 5-year returns for load and no-load mutual funds. ■

EXERCISES

37. Consider again Exercise 32 in Section 2.8. Two independent random samples of applicants to business schools who had and did not have work experience were chosen. Each sample contained 50 applicants. To determine whether there is a difference in the population average test scores, the following hypotheses should be tested:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

Use a 5% level of significance. State the decision rule, the test statistic, and your decision. What implication do these test results have for admissions officers in MBA programs?

38. Use the information in Exercise 33. Suppose that the manufacturer currently uses supplier 2. A change to supplier 1 will be made only if the average lifetime of parts for supplier 1 is greater than the average for supplier 2. Using a 1% level of significance, conduct the appropriate test. State the hypotheses to be tested, the decision rule, the test statistic, and your decision. Assume that the population variances are equal and the basis of the test result, which supplier will the manufacturer choose?

39. Use the information in Exercise 34. Is there a difference in population mean rating scores for the two divisions? State the hypotheses to be tested, the decision rule, the test statistic, and your decision. Assume that the population variances are equal and the populations are normally distributed. Use a 5% level of significance.

40. The 1-year returns for a random sample of 51 load mutual funds and 32 no-load funds were obtained. Test to see if there is any difference between the population average 1-year returns for load and no-load funds. Use a 5% level of significance. State the decision rule, the test statistic, and your decision. Is there a difference in the population averages? Based on the test results, what conclusions do you draw concerning investment in load versus no-load funds? The returns are in a file named RETURNS2. See Exercise 35 for a further description of the data in the file.

41. Let μ_0 = the population average graduation rate average graduation rate for private schools. Suppose that a claim is made that private schools have higher graduation rates, on average, than public schools. Examine the claim by testing the following hypotheses:

$$H_0: \mu_0 - \mu_1 \geq 0$$

ADDITIONAL EXERCISES

42. A university wants to examine starting monthly salaries for its finance and marketing graduates. Independent random samples of 12 finance graduates and 12 marketing graduates are selected from the files of last year's graduates. The following starting salaries are obtained from these people:

Finance	Marketing
1850	1675
2150	1275
1700	1800
1500	2100
2200	2200
1650	2250
2100	1950
2140	1850
1790	2000
1650	1800
2300	2100
2000	2150

Use these data to determine the following:

a. Find the sample mean starting salaries for finance graduates and marketing graduates (separately).

b. Construct a histogram for starting salaries in both finance and marketing.

c. Construct a 95% confidence interval estimate of the population mean starting salary for finance majors. Do the same for marketing majors. Assume that the populations of starting salaries for both groups are normally distributed.

The data are available in the file named SALARY2. The salary data are in column one. The second column indicates to which sample each column one value belongs: 1 = finance; 0 = marketing (in the Excel spreadsheet, the

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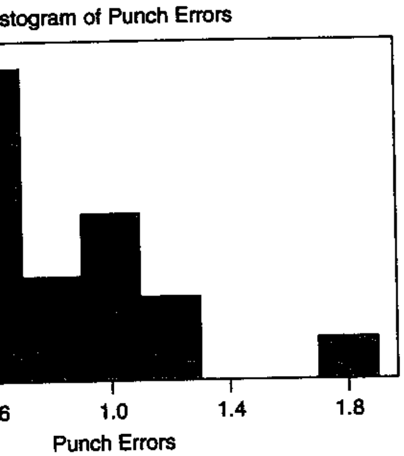
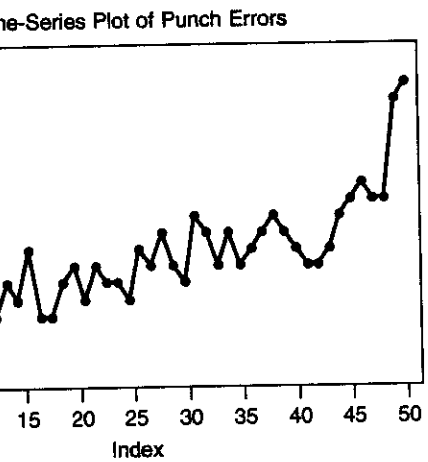




distribution. For example, the probability is 0.9 that there will be no loss; it is 0.02 that there will be a $1000 loss, etc. Note that these losses are per household.

x	$p(x)$
0	0.90
1000	0.02
2000	0.04
3000	0.04

If the insurance company only wants to break even in the long run (dumb company), what should it

52. The filling machine for gallons of milk to be used by school districts in Texas can be set so that it discharges an average of μ ounces of milk per gallon. It is impossible to put exactly 1 gallon of milk in each gallon container due to natural variability in the dispensing machine. The amount of milk discharged is known to have a normal distribution with standard deviation equal to 0.4 ounce. When school officials discover that containers regularly have less than 1 gallon of milk in them, the state gets very upset. So we want to find a setting for μ so that only 1% of the containers of milk will have less than 1 gallon (128 ounces). What value should be used for μ?

53. A computer shop builds PCs from shipments of parts it receives from various suppliers. The number of defective hard drives per shipment is to be modeled as a random variable X. The random variable is assumed to have the following distribution:

x	$P(x)$
0	0.55
1	0.15
2	0.10
3	0.10
4	0.05
5	0.05

a. What is the expected number of defective hard drives per shipment?

b. What is the probability that a shipment will have more than two defective hard drives?

c. What is the probability that a shipment will have four or fewer defective hard drives?

d. What is the probability that a shipment will have more than two but less than five defective hard drives?

e. What is the most likely number of defectives in a shipment?

54. Proctor and Gamble (P&G) has developed a new brand of toothpaste and plan to begin marketing the new toothpaste next month. From test market studies, P&G estimates that demand for the new toothpaste will average 200,000 tubes nationally with a standard deviation of 15,000 tubes during the first year. The *break-even point* for the toothpaste P&G will need to sell 180,000 tubes to break even in the first year. Assume that demand for the new toothpaste follows a normal distribution with mean 200,000 and standard deviation 15,000. What is the probability that P&G will sell at or above the break-even point during the first year?

55. High-Tech, Inc. produces an electronic component, GS-7, that has an average life span of 1000 hours. The life span is normally distributed with a standard deviation of 25 hours. The company is considering how long a warranty to place on the component. If it wants to replace no more than 10% of the components for free, how many hours should it guarantee the components would last under the warranty?

56. A bank reports that the population of its demand deposit balances has a population mean of $7500 and a population standard deviation of $1000. An auditor refuses to certify the bank's claim and takes a random sample of 100 account balances. He will certify the bank's report only if the sample mean is no more than $125 above or below the bank's stated population mean.

a. Assuming the bank's reported figures are correct, what is the probability that the auditor will certify the bank's report?

b. What would be the probability that the auditor will certify the bank's report if the reported population mean were $9000 rather than $7500 (assuming the standard deviation is the same)?

57. Researchers at American Airlines and Sabre have developed a system to coordinate scheduling, yield management, and pricing decisions (*ORMS Today*, August 2000, pp. 36–44). In one of their examples they set up the following scenario: "Consider the case of a single flight leg with one fare class. For this example, we assume that the passenger demand follows a normal distribution with a mean of 125 passengers and a coefficient of variation of 0.3. The capacity of the leg is assumed to be 150 seats." The coefficient of variation is defined as the standard deviation divided by the mean. What is the probability that demand will exceed the flight capacity?

58. A city engineer recorded the number of vehicles

determine the percentage of days that more than 425 vehicles used the intersection. Suppose the mean for the data was 375 vehicles per day and the standard deviation was 25 vehicles. What can you say about the percentage of days that more than 425 vehicles used the intersection? Assume the distribution of the data is a normal distribution.

59. Our company manufactures a certain electronic component for use in the new F-22 fighter being built by Lockheed. We want to test the reliability of the components, and we think testing a random sample of 50 components will be sufficient. The population standard deviation of the lives of the components is known to be 10 hours. If we use a random sample of 50 components, what is the probability that our sample mean will be within (plus or minus) 2 hours of the true population average lifetime of the components?

60. **Process Capability.** Specification limits for a particular characteristic of a product indicate the values at which the product will operate properly. Specification limits are set by engineering and are not determined by the data. The data must be examined to determine whether the product is within specifications. If the product is within specification limits, the process generating the product is often referred to as capable. For example, assume we have a product with an upper specification limit of USL = 4.25 and a lower specification limit of LSL = 3.0. As long as our product falls within these limits our buyers will be happy. We now investigate the process used to manufacture the product and find that the process mean is 4 and the process standard deviation is 0.2. Assuming the process is normally distributed, what percentage of items will fall outside the specification limits? Should we make changes in the process? If so, what would you suggest?

61. By law, a manufacturer of a food product is required to list Food and Drug Administration (FDA) estimates of the contents of the packaged product. Suppose the FDA wants to estimate the mean sugar content (by weight) in 16-ounce boxes of "Disney Chocolate Mud and Bugs" cereal. The FDA randomly selects 200 boxes of Disney Chocolate Mud and Bugs, measures the sugar content in each, and computes a 95% confidence interval estimate of the average sugar content to be (3.2, 4.5).

The manufacturer plans to use the interval to claim that 95% of all boxes of Disney Chocolate Mud and Bugs cereal have sugar content weights between 3.2 and 4.5 ounces. Is this a correct interpretation of the interval? Justify your answer.

62. Suppose a sample of $n = 100$ items is randomly chosen. The following hypotheses are to be tested:

$$H_0{:}\mu \leq 10$$
$$H_a{:}\mu > 10$$

A p value of 0.0409 for the test is obtained. If the population standard deviation is 5, what would the value of the sample mean have to be in order to achieve the stated p value?

USING THE COMPUTER

The Using the Computer section in each chapter describes how to perform the computer analyses in the chapter using Excel, MINITAB, and SAS. For further detail on Excel, MINITAB, and SAS, see Appendix C.

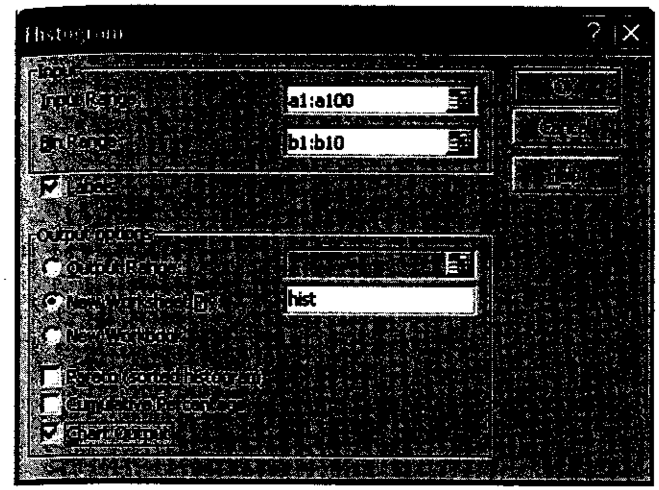
EXCEL

Descriptive Statistics

TOOLS: DATA ANALYSIS: HISTOGRAM

Click Tools, then Data Analysis, and then Histogram. Histogram creates a frequency distribution and a histogram. In the histogram dialog box (see Figure 2.34), fill in the

FIGURE 2.34 Excel Histogram Dialog Box.



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Pareto chart emphasizes those complaints by lis…
centage option constructs a histogram but super…
represents the cumulative percentage (sometim…
quests Excel to construct a histogram in addition…
Once you have the options set as you want,…

TOOLS: DATA ANALYSIS: DESCRIPTIVE STATISTICS

Click Tools, then Data Analysis, and then De…
generates a variety of descriptive statistics (see…
tistics dialog box (see Figure 2.35), fill in the i…
descriptive statistics are desired. Typically, this…
tion is available to indicate whether it is in a co…
tion and click Summary statistics. Click Conf…