In class midterm: Oct. 16

Homework Assignments (#2)
(Due Tuesday, 09/30)

Note: The problem numbers in parentheses are from the sixth edition.

Chapter 8: # 5, 9, 17, 20, 23, 35, 40, 39 (Note: The wording in the question is not precise. The “random sample of 1050” referred to in the question was chosen by procedures developed by the court. The issue here was whether these procedures actually produced a random sample (as they should have), or merely a sample. Swain’s attorneys – and the solutions given in the back of our book – used a one-sided test. Suppose you were the defense lawyer, representing the state of Alabama. How would you argue that a two-sided test was more appropriate for this situation? Would it have made any substantive difference to the final evaluation if a two-sided test had been used?)

D. The intended octane rating of the gasoline produced by the company in Problem B, (HW 1), is 87.

(a) Use the data of problem B to test (at level $\alpha = 0.05$) whether the company’s mean octane rating for the day in question was different from 87.

(b) Actually, the company does not claim that its “regular” grade gasoline has an octane rating of exactly 87. Instead it claims that the octane rating is at least 87. Perform a test (at level $\alpha = 0.05$) designed to verify that the company’s claim is correct.

E. While imprisoned by the Germans in World War II, the English mathematician John Kerrich tossed his only coin 10,000 times and obtained 5,067 heads. Let $p$ denote the probability of a head on a single toss. We wish to check whether the data are consistent with the hypothesis that the coin was fair.

(a) Set up the statistical hypotheses. Why should the alternative be two-sided even though Kerrich obtained more heads than tails on his 10,000 tosses?

(b) Does your test reject the null hypothesis at $\alpha = 0.05$?

(c) Find a 95% CI for the proportion of heads for Kerrich’s coin. Does this CI contain $p = \frac{1}{2}$?

(d) On his first 1,000 flips Kerrich obtained 574 heads. Does this information alter your conclusions in (b)?

More problems from Chapter 8: #44, 47 (Also, what would be the P-value in each case if the problem had been to test $H_0: \mu = 30$ versus $H_a: \mu > 30?$), 53 (55), 59 (61) (assume normality), 60 (62), 69 (71), 72 (73), 73 (75), and
F. What is the P-value for the situation in problem E(a), above? How is this P-value related to your conclusion in problem E(b)? How is the confidence interval in problem E(c) related to the conclusion of the test in E(b)?

G. How is the CI in problem B related to the result of the hypothesis test in problem D(a)?

H. A blood test intended to identify patients at “high risk” of cardiac disease gave positive results on 50 out of 60 known cardiac patients, but also on 16 out of 200 known normal patients.

(a) Find a 90% CI for the sensitivity of the test, which is defined as the probability that a “non-normal” patient is correctly identified. (Here, “non-normal” should be interpreted as “cardiac”.) Would a test of $H_0: p = 0.8$ versus $H_a: p \neq 0.8$ accept or reject at $\alpha = 0.1$?

(b) Find a 90% CI for the specificity of the test, which is defined as the probability that a normal patient is correctly identified. Would a test of $H_0: p = 0.75$ versus $H_a: p \neq 0.75$ accept or reject at $\alpha = 0.1$? (Note: It is usually desirable to have sensitivity $\geq$ specificity. Why? But that appears not to be the case for this blood test.)

I. Many additional cardiac patients were then given the blood test in problem F and the sensitivity of that test was determined to be almost exactly 80%. A new testing method has been derived that supplements the blood test with information based on electrocardiogram measurements. This new method was tested on 50 cardiac patients and gave correct results for 46 of them. Is this convincing evidence that the new method has a higher sensitivity than the old method?

(a) Set up the hypotheses to validate the theory that the sensitivity of the new method is better than that of the old. (It is assumed that the sensitivity cannot be worse.)

(b) Give the P-value for this test corresponding to the observed data. Does a level $\alpha = 0.05$ test reject or not-reject? Did you use the large-sample (normal) approximation or the small-sample (binomial) procedure? Why? Would it make a difference in your result?