Homework Solution (#2)

Chapter 8: #5, 9, 17, 20, 23, 35, 40, 39, D, E, 44, 47, 53, 59, 60, 69, 72, 73, F, G, H and I

Chapter 8. Test of Hypothesis Based on a Single Sample

8.5 (Page 319, old edition; Page 324, new edition)

Let σ denote the population standard deviation. The appropriate hypotheses are $H_o : \sigma = .05$ vs. $H_a : \sigma < .05$. With this formulation, the burden of proof is on the data to show that the requirement has been met (the sheaths will not be used unless H_o can be rejected in favor of H_a. Type I error: Conclude that the standard deviation is < .05 mm when it is really equal to .05 mm. Type II error: Conclude that the standard deviation is .05 mm when it is really < .05.

8.9 (Page 320, old edition; Page 325, new edition)

a. R_1 is most appropriate, because x either too large or too small contradicts p = .5 and supports p $\neq$. 5

b. A type I error consists of judging one of the tow candidates favored over the other when in fact there is a 50-50 split in the population. A type II error involves judging the split to be 50-50 when it is not.

c. X has a binomial distribution with n = 25 and p = 0.5. $\alpha =$ P (type I error) = $P(X \leq 7 or X \geq 18$ when X ~ Bin (25, .5)) = B(7; 25,.5) + 1 – B(17; 25,.5) = .044

d. $\beta(.4) = P(8 \leq X \leq 17$ when p = .4) = B(17; 25,.5) – B(7, 25,.4) = 0.845, and $\beta(.6) = 0.845$ also. $\beta(.3) = B(17;25,.3) - B(7;25,.3) = .488 = \beta(.7)$

e. x = 6 is in the rejection region R_1 , so H_o is rejected in favor of H_a.

8.17 (Page 332, old edition; Page 337, new edition)

a. $z = \dfrac{20{,}960 - 20{,}000}{1500/\sqrt{16}} = 2.56 > 2.33$ so reject H_o.

b. $\beta(20{,}500) : \Phi\left(2.33 + \dfrac{20{,}000 - 20{,}500}{1500/\sqrt{16}}\right) = \Phi(1.00) = .8413$

c. $\beta(20{,}500) = .05 : n = \left[\dfrac{1500(2.33 + 1.645)}{20{,}000 - 20{,}500}\right]^2 = 142.2$, so use n = 143

d. $\alpha = 1 - \Phi(2.56) = .0052$

8.20 (Page 332, old edition; Page 337, new edition)

With H_o: $\mu = 750$, and H_a: $\mu < 750$ and a significance level of .05, we reject H_o if z < -1.645; z = -2.14 < -1.645, so we reject the null hypothesis and do not continue with the purchase. At a significance level of .01, we reject H_o if z < -2.33; z = -2.14 > -2.33, so we don't reject the null hypothesis and thus continue with the purchase.

8.23 (Page 332, old edition; Page 337, new edition)

H_o: $\mu = 360$ vs. H_a: $\mu > 360$; $t = \frac{\bar{x} - 360}{s/\sqrt{n}}$; reject H_o if $t > t_{.05,25} = 1.708$;

$t = \frac{370.69 - 360}{24.36/\sqrt{26}} = 2.24 > 1.708$. Thus H_o should be rejected. There appears to be a contradiction of the prior belief.

8.35 (Page 338, old edition; Page 343, new edition)

(1). Parameter of interest: p = true proportion of cars in this particular county passing emissions testing on the first try.

(2). H_o: p = .70

(3). H_a: p ≠ .70

(4). $z = \frac{\hat{p} - p_o}{\sqrt{p_o(1-p_o)/n}} = \frac{\hat{p} - .70}{\sqrt{.70(.30)/n}}$

(5). Either z ≥ 1.96 or z ≤ -1.96

(6). $z = \frac{156/200 - .70}{\sqrt{.70(.30)/200}} = 2.469$

(7). Reject H_o. The data indicates that the proportion of cars passing the first time on emission testing or this county differs from the proportion of cars passing statewide.

8.40 (Page 339, old edition; Page 344, new edition)

a. P = true proportion of current customers who qualify. H_o: p = .05 vs. H_a: p ≠ .05, $z = \frac{\hat{p} - .05}{\sqrt{.05(.95)/n}}$, reject H_o if z ≥ 2.58 or z ≤ -2.58. $\hat{p} = .08$. So

$z = \frac{.03}{.00975} = 3.07 \geq 2.58$, so H_o is rejected. The company's premise is not correct.

b. $\beta(.10) = \Phi\left[\frac{.05 - .10 + 2.58\sqrt{.05(.95)/500}}{\sqrt{.10(.90)/500}}\right] = \Phi(-1.85) = .0332$

8.39 (Page 339, old edition; Page 344, new edition)

Let p denote the true proportion of those called to appear for service that is black. We wish to test H_o: p = .25 vs. H_a: p < .25. We use $z = \frac{\hat{p} - .25}{\sqrt{.25(.75)/n}}$, with the rejection region z ≤ $-z_{.01}$ = -2.33. We calculate $\hat{p} = \frac{177}{1050} = .1686$, and

$z = \frac{.1686 - .25}{.0134} = -6.1$. Because –6.1 < -2.33, H_o is rejected. A conclusion that discrimination exists is very compelling.

Suppose you were the defense lawyer, then before looking at the data, we wish to test the following two-sided hypothesis: H_o: p = .25 vs. H_a: p ≠ .25 to avoid a bias mind testing.
Then $z_{.005}$ = -2.57. Because –6.1 < -2.33, H_o is rejected. It doesn't make any substantive difference to the final evaluation if a two-sided test had been used.

D. n = 20, $\bar{x} = 87.40$, s = .52;

a. H_o: μ =87 vs. H_a: $\mu \neq$87; $t = \frac{\bar{x}-87}{s/\sqrt{n}}$; reject H_o if $|t| > t_{.025,19} = 2.093$;

$t = \frac{87.4-87}{.52/\sqrt{20}} = 3.440 > 2.093$. Thus H_o should be rejected. So the mean octane rating of the day's production was significantly different from 87.

b. H_o: μ =87 vs. H_a: μ >87; $t = \frac{\bar{x}-87}{s/\sqrt{n}}$; reject H_o if $t > t_{.05,19} = 1.729$;

$t = \frac{87.4-87}{.52/\sqrt{20}} = 3.440 > 1.729$. Thus H_o should be rejected. So the mean octane rating of the day's production was at least 87.

E. n=10000, $\hat{p} = \frac{5067}{10000}$;

a. H_o: p = .50 vs. H_a: p ≠ .50. Before obtaining the observations, he would set up the zero hypotheses to test whether the coin is a biased one.

b. $z = \frac{5067/10000-.50}{\sqrt{.5(1-.5)/10000}} = 1.34 < |z_{.025}| = 1.96$. So don't reject zero hypotheses.

c. The 95% confidence interval for p is:

$.5067 \pm 1.96\sqrt{\frac{(.5067)(1-.5067)}{10000}} = (.497,.516)$. It contains p=0.5.

d. $z = \frac{574/1000-.50}{\sqrt{.5(1-.5)/1000}} = 4.68 > |z_{.025}| = 1.96$. So reject zero hypotheses!

8.44 (Page 345, old edition; Page 350, new edition)

Using α = .05, H_o should be rejected whenever p-value < .05.

a. P-value = .001 < .05, so reject H_o

b. 0.021 < .05, so reject H_o.

c. 0.078 is not < .05, so don't reject H_o.

d. 0.047 < .05, so reject H_o (a close call).

e. 0.148 > .05, so H_o can't be rejected at level .05.

8.47 (Page 346, old edition; Page 351, new edition)

a. 0.0358

b. 0.0802

c. 0.5824

d. 0.1586

e. 0

8.53 (Page 346, old edition),[8.55 (Page 352, new edition)]

The hypotheses to be tested are H_o: $\mu = 25$ vs. H_a: $\mu > 25$, and H_o should be rejected if $t \geq t_{.05,12} = 1.782$. The computed summary statistics are $\bar{x} = 27.923$, $s = 5.619$, so $\frac{s}{\sqrt{n}} = 1.559$ and $t = \frac{2.923}{1.559} = 1.88$. Because $t_{.025,12} = 2.179 > 1.88 > t_{.05,12} = 1.782$, the p-value is between .025 and .05, so H_o is rejected at level .05.

8.59 (Page 350, old edition), [8.61(Page 356, new edition)]

Because n = 50 is large, we use a z test here, rejecting H_o: $\mu = 3.2$ in favor of H_a: $\mu \neq 3.2$ if either $z \geq z_{.025} = 1.96$ or $z \leq -1.96$. The computed z value is $z = \frac{3.05 - 3.20}{.34/\sqrt{50}} = -3.12$. Since $-3.12 \leq -1.96$, H_o should be rejected in favor of H_a.

8.60 (Page 350, old edition), [8.62(Page 356, new edition)]

Here we assume that thickness is normally distributed, so that for any *n* a t test is appropriate, and use Table A.17 to determine n. We wish $\pi(3) = .95$ when

$d = \frac{|3.2 - 3|}{.3} = .667.$

$n = [\frac{\sigma(z_{\alpha/2} + z_{\beta})}{\mu_0 - \mu'}]^2 = [(1.96 + 1.645)/.667]^2 = 30$

So n = 50 is too large.

8.69 (Page 352, old edition), [8.71 (Page 357, new edition)]

a. With H_o: p = $1/75$ vs. H_o: p $\neq 1/75$, we reject H_o if either $z \geq 1.96$ or $z \leq -1.96$. With $\hat{p} = \frac{16}{800} = .02$, $z \frac{.02 - .01333}{\sqrt{\frac{.01333(.98667)}{800}}} = 1.645$, which is not in either rejection region. Thus, we fail to reject the null hypothesis. There is not evidence that the incidence rate among prisoners differs from that of the adult population. The possible error we could have made is a type II.

b. P-value = $2[1 - \Phi(1.645)] = 2[.05] = .10$. Yes, since .10 < .20, we could reject H_o.

8.72 (Page 352, old edition)

Let p = the true proportion of mechanics who could identify the problem. Then the appropriate hypotheses are H_o: p = .75 vs. H_a: p < .75, so a lower-tailed test should be used. With p_o= .75 and $\hat{p} = \frac{42}{72} = .583$, z = -3.28 and $P = \Phi(-3.28) = .0005$. Because this p-value is so small, the data argues strongly against H_o, so we reject it in favor of H_a.

8.73 (Page 352, old edition), [8.75 (Page 357, new edition)]

We wish to test H_o: $\lambda = 4$ vs. H_a: $\lambda > 4$ using the test statistic $z = \frac{\bar{x} - 4}{\sqrt{4/n}}$. For the given sample, n = 36 and $\bar{x} = \frac{160}{36} = 4.444$, so $z = \frac{4.444 - 4}{\sqrt{4/36}} = 1.33$. At level .02, we reject H_o if $z \ge z_{.02} \doteq 2.05$ (since $1 - \Phi(2.05) = .0202$). Because 1.33 is not ≥ 2.05, H_o should not be rejected at this level.

F.

P-value in problem E(a) is 2(.0901)=.180. Since .180>.05, so don't reject H_o in problem E(b). Since two-sided 95% confidence interval contains point .50, so don't reject H_o in problem E(b).

G.

Since 95% confidence interval: (87.15,87.64) (Problem B) doesn't contain point 87, so reject H_o in problem in D(a).

H.

a. n=60, $\hat{p}$ =0.833. So the 90% confidence interval for the **sensitivity** of the test:

$$\frac{.833 + \frac{(1.65)^2}{2(60)} \pm 1.65\sqrt{\frac{(.833)(.167)}{60} + \frac{(1.65)^2}{4(60)^2}}}{1 + \frac{(1.65)^2}{60}} = (.740,.898)$$

Since the 90% confidence interval contains .80, so don't reject H_o.

b. n=200, $\hat{p}$ =0.920. So the 90% confidence interval for the **specificity** of the test:

$$\frac{.920 + \frac{(1.65)^2}{2(200)} \pm 1.65\sqrt{\frac{(.920)(.080)}{200} + \frac{(1.65)^2}{4(200)^2}}}{1 + \frac{(1.65)^2}{60}} = (.882,.946)$$

Since the 90% confidence interval doesn't contain .75, so reject H_o.

I.

a. Denote p as the true sensitivity of the new method. Then set the hypotheses:
H_o: p = 0.8 vs. H_o: p > 0.8;

b. $z = \frac{46/50 - .8}{\sqrt{.8(1-.8)/50}} = 2.12$. The corresponding P-value is .017 < .05. So reject zero hypotheses. We use the large-sample approximation. Since n=50 is pretty large.