Homework Solution (#3)

Chapter 9: #: 2, 7, 12, 19, 25, 27, 36, 41, 42, 69, 47, 48, 49, 56, 67, 80 and 86

Chapter 9. Test of Hypothesis Based on a Single Sample

9.2 (Page 364 old edition; Page 370, new edition)

The test statistic value is $z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$, and H_o will be rejected if either $z \geq 1.96$ or $z \leq -1.96$. We compute $z = \frac{42{,}500 - 40{,}400}{\sqrt{\frac{2200^2}{45} + \frac{1900^2}{45}}} = \frac{2100}{433.33} = 4.85$. Since 4.85 > 1.96, reject H_o and conclude that the two brands differ with respect to true average tread lives.

9.7 (Page 364 old edition; Page 370, new edition)

(1). Parameter of interest: $\mu_1 - \mu_2 =$ the true difference of means for males and females on the Boredom Proneness Rating. Let $\mu_1 =$ men's average and $\mu_2 =$ women's average.

(2). H_o: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 > 0$

(3). $z = \frac{(\bar{x} - \bar{y}) - \Delta_o}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}} = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$

(4). Rejection Region: $z \geq 1.645$

(5). $z = \frac{(10.40 - 9.26) - \Delta_o}{\sqrt{\frac{4.83^2}{97} + \frac{4.68^2}{148}}} = 1.83$

(6). Reject H_o. The data indicates the Boredom Proneness Rating is higher for males than for females.

9.12 (Page 365, old edition), [9.13(Page372, new edition)]

$\sigma_1 = \sigma_2 = .05$, d = .04, $\alpha = .01, \beta = .05$, and the test is one-tailed, so

$$n = \frac{(.0025 + .0025)(2.33 + 1.645)^2}{.0016} = 49.38$$, so use n = 50.

9.19 (Page 371, old edition; Page 378, new edition)

For the given hypotheses, the test statistic $t = \dfrac{115.7 - 129.3 + 10}{\sqrt{\frac{5.03^2}{6} + \frac{5.38^2}{6}}} = \dfrac{-3.6}{3.007} = -1.20$,

and the d.f. is $\nu = \dfrac{(4.2168 + 4.8241)^2}{\dfrac{(4.2168)^2}{5} + \dfrac{(4.8241)^2}{5}} = 9.96$, so use d.f. = 9. We will reject H_o if $t \le -t_{.01,9} = -2.764$; since –1.20 > -2.764, we don't reject H_o.

9.25 (Page 372, old edition; Page 379, new edition)

We calculate the degrees of freedom $\nu = \dfrac{\left(\frac{5.5^2}{28} + \frac{7.8^2}{31}\right)^2}{\dfrac{\left(\frac{5.5^2}{28}\right)^2}{27} + \dfrac{\left(\frac{7.8^2}{31}\right)^2}{30}} = 53.95$, or about 54 (normally we would round down to 53, but this number is very close to 54 – of course for this large number of d.f., using either 53 or 54 won't make much difference in the critical t value) so the desired confidence interval is $(91.5 - 88.3) \pm 1.68\sqrt{\frac{5.5^2}{28} + \frac{7.8^2}{31}} = 3.2 \pm 2.931 = (.269, 6.131)$. Because 0 does not lie inside this interval, we can be reasonably certain that the true difference $\mu_1 - \mu_2$ is not 0 and, therefore, that the two population means are not equal. For a 95% interval, the t value increases to about 2.01 or so, which results in the interval 3.2 ± 3.506. Since this interval does contain 0, we can no longer conclude that the means are different if we use a 95% confidence interval.

9.27 (Page 372 old edition; Page 379, new edition)

The approximate degrees of freedom for this estimate are v= $\dfrac{\left(\frac{11.3^2}{6} + \frac{8.3^2}{8}\right)^2}{\dfrac{\left(\frac{11.3^2}{6}\right)^2}{5} + \dfrac{\left(\frac{8.3^2}{8}\right)^2}{7}}$

$= \dfrac{893.59}{101.175}$ =8.83, which we round down to 8, so $t_{.025,8} = 2.306$ and the desired interval is (40.3-21.4) ±2.306 $\sqrt{\frac{11.3^2}{6} + \frac{8.3^2}{8}}$ $= 18.9 \pm 2.306(5.4674)$ $= 18.9 \pm 12.607 = (6.3, 31.5)$. Because 0 is not contained in this interval, there is strong evidence that $\mu_1 - \mu_2$ is not 0; i.e., we can conclude that the population means are not equal. Calculating a confidence interval for $\mu_2 - \mu_1$ would change only the order of subtraction of the sample means, but the standard error calculation would give the same result as before. Therefore, the 95% interval estimate of $\mu_2 - \mu_1$ would be (-31.5, -6.3), just the negatives of the endpoints of the original interval. Since 0 is not in this interval, we reach exactly the same conclusion as before; the population means are not equal.

9.36 (Page 381 old edition; Page 388, new edition)

$\bar{d} = 7.25$, $s_D = 11.8628$

(1). Parameter of Interest: $\mu_D =$ true average difference of breaking load for fabric in unabraded or abraded condition.

(2). $H_0 : \mu_D = 0$ vs. $H_a : \mu_D > 0$

(3). $t = \dfrac{\bar{d} - \mu_D}{s_D / \sqrt{n}} = \dfrac{\bar{d} - 0}{s_D / \sqrt{n}}$

(4). Rejection Region: $t \geq t_{.01,7} = 2.998$

(5). $t = \dfrac{7.25 - 0}{11.8628 / \sqrt{8}} = 1.73$

(6). Fail to reject H_o. The data does not indicate a difference in breaking load for the two fabric load conditions.

9.41 (Page 383 old edition; Page 390, new edition)

We test $H_0 : \mu_d = 0$ vs. $H_a : \mu_d > 0$. With $\bar{d} = 7.600$, and $s_d = 4.178$,
$t = \dfrac{7.600 - 5}{4.178 / \sqrt{9}} = \dfrac{2.6}{1.39} = 1.87 \approx 1.9$. With degrees of freedom n – 1 = 8, the corresponding p-value is P (t > 1.9) = .047. We would reject H_o at any alpha level greater than .047. So, at the typical significance level of .05, we would (barely) reject H_o, and conclude that the data indicates that the higher level of illumination yields a decrease of more than 5 seconds in true average task completion time.

9.42 (Page 383 old edition; Page 390, new edition)

(1). Parameter of interest: μ_d denotes the true average difference of spatial ability in brothers exposed to DES and brothers not exposed to DES. Let $\mu_d = \mu_{exposed} - \mu_{unexposed}$.

(2). $H_0 : \mu_D = 0$ vs. $H_a : \mu_D < 0$

(3). $t = \dfrac{\bar{d} - \mu_D}{s_D / \sqrt{n}} = \dfrac{\bar{d} - 0}{s_D / \sqrt{n}}$

(4). Rejection Region: P-value < .05, df = 8

(5). $t = \dfrac{(12.6 - 13.7) - 0}{0.5} = -2.2$, with corresponding p-value .029. (Table A.8)

(6). Reject H_o. The data supports the idea that exposure to DES reduces spatial ability.

9.69 (Page 397 old edition; Page 404, new edition)

The center of any confidence interval for $\mu_1 - \mu_2$ is always $\bar{x}_1 - \bar{x}_2$, so $\bar{x}_1 - \bar{x}_2 = \dfrac{-473.3 + 1691.9}{2} = 609.3$. Furthermore, half of the width of this interval is $\dfrac{1691.9 - (-473.3)}{2} = 1082.6$. Equating this value to the expression on the right of the 95% confidence interval formula, $1082.6 = (1.96)\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$, we find $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \frac{1082.6}{1.96} = 552.35$. For a 90% interval, the associated z value is 1.645, so the 90% confidence interval is then $609.3 \pm (1.645)(552.35) = 609.3 \pm 908.6$ $= (-299.3, 1517.9)$.

9.47 (Page 390 old edition; Page 397, new edition)

H_o will be rejected if $z \le -z_{.01} = -2.33$. With $\hat{p}_1 = .150$, and $\hat{p}_2 = .300$, $\hat{p} = \frac{30+80}{200+600} = \frac{210}{800} = .263$, and $\hat{q} = .737$. The calculated test statistic is $z = \frac{.150 - .300}{\sqrt{(.263)(.737)\left(\frac{1}{200} + \frac{1}{600}\right)}} = \frac{-.150}{.0359} = -4.18$. Because $-4.18 \le -2.33$, H_o is rejected; the proportion of those who repeat after inducement appears lower than those who repeat after no inducement.

9.48 (Page 390 old edition; Page 397, new edition)

a. H_o will be rejected if $z \le -1.96$ or $z \ge 1.96$. With $\hat{p}_1 = \frac{63}{300} = .2100$, and $\hat{p}_2 = \frac{75}{180} = .4167$, $\hat{p} = \frac{63+75}{300+180} = .2875$, z= $\frac{.2100 - .4167}{\sqrt{(.2875)(.7125)\left(\frac{1}{300} + \frac{1}{180}\right)}}$ $= \frac{-.2067}{.0427} = -4.84$. Since-4.84<-1.96, H_o is rejected.

b. $\bar{p} = .275$ and $\sigma = .0432$, so power =

$$1 - \Phi\left(\frac{[(1.96)(.0421) + .2]}{.0432}\right) + \Phi\left(\frac{[(-1.96)(.0421) + .2]}{.0432}\right) = 1 - \Phi(6.57) + \Phi(2.73)$$

$= .9968$.

9.49 (Page 390 old edition; Page 397, new edition)

(1). Parameter of interest: $p_1 - p_2$ = true difference in proportions of those responding to two different survey covers. Let p_1 = Plain, p_2 = Picture.

(2). $H_0 : p_1 - p_2 = 0$ vs. $H_a : p_1 - p_2 < 0$

(3). $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}}$

(4). Reject H_o if p-value < .10

(5). $z = \frac{\frac{104}{207} - \frac{109}{213}}{\sqrt{\left(\frac{213}{420}\right)\left(\frac{207}{420}\right)\left(\frac{1}{207} + \frac{1}{213}\right)}} = -.1910$; p-value = .4247

(6). Fail to Reject H_o. The data does not indicate that plain cover surveys have a lower response rate.

9.56 (Page 392 old edition; Page 398, new edition)

Using $p_1 = q_1 = p_2 = q_2 = .5$, $L = 2(1.96)\sqrt{\left(\frac{.25}{n}+\frac{.25}{n}\right)} = \frac{2.7719}{\sqrt{n}}$, so L = .1 requires n = 769.

9.67 (Page 397 old edition; Page 404, new edition)

Let p_1 = true proportion of returned questionnaires that included no incentive; p_2 = true proportion of returned questionnaires that included an incentive. The hypotheses are $H_0 : p_1 - p_2 = 0$ vs. $H_0 : p_1 - p_2 < 0$. The test statistic is $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m}+\frac{1}{n}\right)}}$. $\hat{p}_1 = \frac{75}{110} = .682$, and $\hat{p}_2 = \frac{66}{98} = .673$. At this point we notice that since $\hat{p}_1 > \hat{p}_2$, the numerator of the z statistic will be > 0, and since we have a lower tailed test, the p-value will be > .5. We fail to reject H_o. This data does not suggest that including an incentive increases the likelihood of a response.

9.80 (Page 399 old edition; Page 406, new edition)

The relevant hypotheses would be $\mu_M = \mu_F$ versus $\mu_M \neq \mu_F$ for both the distress and delight indices. The reported p-value for the test of mean differences on the distress index was less than 0.001. This indicates a statistically significant difference in the mean scores, with the mean score for women being higher. The reported p-value for the test of mean differences on the delight index was > 0.05. This indicates a lack of statistical significance in the difference of delight index scores for men and women.

9.86 (Page 400, old edition) [9.88 (Page 370, new edition)]

$H_0 : \mu_1 - \mu_2 = 0$ is tested against $H_a : \mu_1 - \mu_2 \neq 0$ using the two-sample t test, rejecting H_o at level .05 if either $t \geq t_{.025,15} = 2.131$ or if $t \leq -2.131$. With $\bar{x} = 11.20$, $s_1 = 2.68$, $\bar{y} = 9.79$, $s_2 = 3.21$, and m = n = 8, $s_p = 2.96$, and t = .95, so H_o is not rejected. In the situation described, the effect of carpeting would be mixed up with any effects due to the different types of hospitals, so no separate assessment could be made. The experiment should have been designed so that a separate assessment could be obtained (e.g., a randomized block design).