Homework Solution (#4)

Chapter 10: 4, 9 26,29,42 and 46

Chapter 10. The Analysis of Variance

10.4 (new edition: Page 420; old edition: Page 412)

$x_{\bullet\bullet} = IJ\bar{x}_{\bullet\bullet} = 32(5.19) = 166.08$, so $SST = 911.91 - \frac{(166.08)^2}{32} = 49.95$.

$SSTr = 8\left[(4.39-5.19)^2 + ... + (6.36-5.19)^2\right] = 20.38$, So

$SSE = 49.95 - 20.38 = 29.57$. Then $f = \frac{20.38/3}{29.57/28} = 6.43$. Since

$6.43 \ge F_{.05,2,28} = 2.95$, $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ is rejected at level .05. There are differences between at least two average flight times for the four treatments.

10.9 (new edition: Page 421; old edition: Page 413)

The summary quantities are $x_{1\bullet} = 34.3$, $x_{2\bullet} = 39.6$, $x_{3\bullet} = 33.0$, $x_{4\bullet} = 41.9$,

$x_{\bullet\bullet} = 148.8$, $\Sigma\Sigma x_{ij}^2 = 946.68$, so $CF = \frac{(148.8)^2}{24} = 922.56$,

$SST = 946.68 - 922.56 = 24.12$, $SSTr = \frac{(34.3)^2 + ... + (41.9)^2}{6} - 922.56 = 8.98$,

$SSE = 24.12 - 8.98 = 15.14$.

Source	Df	SS	MS	F
Treatments	3	8.98	2.99	3.95
Error	20	15.14	.757	
Total	23	24.12		

Since $3.10 = F_{.05,3,20} < 3.95 < 4.94 = F_{.01,3,20}$, $.01 < p - value < .05$ and H_o is rejected at level .05.

10.26 (new edition: Page 437; old edition: Page 429)

a

i:	1	2	3	4	5	6	
J_I:	4	5	4	4	5	4	
$x_{i\bullet}$:	56.4	64.0	55.3	52.4	85.7	72.4	$x_{\bullet\bullet} = 386.2$
$\bar{x}_{i\bullet}$:	14.1	12.8	13.8	13.1	17.1	18.1	$\Sigma\Sigma x_j^2 = 5850.20$

Thus SST = 113.64, SSTr = 108.19, SSE = 5.45, MSTr = 21.64, MSE = .273, f = 79.3. Since $79.3 \ge F_{.01,5,20} = 4.10$, $H_0 : \mu_1 = ... = \mu_6$ is rejected.

b The modified Tukey intervals are as follows: (The first number is $\bar{x}_{i\bullet} - \bar{x}_{j\bullet}$ and the second is

$$W_{ij} = Q_{.05,I,I(\sum_i J_i)} \cdot \sqrt{\frac{MSE}{2}\left(\frac{1}{J_i}+\frac{1}{J_j}\right)} = 4.45 \cdot \sqrt{\frac{MSE}{2}\left(\frac{1}{J_i}+\frac{1}{J_j}\right)}\text{.)}$$

Pair	Interval	Pair	Interval	Pair	Interval
1,2	1.30 ± 1.1	2,3	-1.03 ± 1.1	3,5	$-3.31 \pm 1.1*$
1,3	$.27 \pm 1.16$	2,4	$-.30 \pm 1.1$	3,6	$-4.27 \pm 1.16*$
1,4	1.00 ± 1.16	2,5	$-4.34 \pm 1.04*$	4,5	$-4.04 \pm 1.16*$
1,5	$-3.04 \pm 1.1*$	2,6	$-5.30 \pm 1.1*$	4,6	$-5.00 \pm 1.1*$
1,6	$-4.00 \pm 1.16*$	3,4	$.37 \pm 1.16$	5,6	$-.96 \pm 1.1$

Asterisks identify pairs of means that are judged significantly different from one another.

c The 95% t confidence interval is $\Sigma c_i \bar{x}_{i\bullet} \pm t_{.025,20}\sqrt{MSE\left(\Sigma \frac{c_i^2}{J_i}\right)}$.

$\Sigma c_i \bar{x}_{i\bullet} = \frac{1}{4}\bar{x}_{1\bullet} + \frac{1}{4}\bar{x}_{2\bullet} + \frac{1}{4}\bar{x}_{3\bullet} + 14\bar{x}_{4\bullet} - 12\bar{x}_{5\bullet} - \frac{1}{2}\bar{x}_{6\bullet} = -4.16$, $\Sigma \frac{c_i^2}{J_i} = .1719$, MSE = .273, $t_{.025,20} = 2.086$. The resulting interval is $-4.16 \pm 2.086\sqrt{(.273)(.1719)} = -4.16 \pm .45 = (-4.61, -3.71)$. The interval in the answer section is a Scheffe' interval, and is substantially wider than the t interval.

10.29 (new edition: Page 437, old edition: Page 429)

$$E(SSTr) = E\left(\sum_i J_i \bar{X}_{i\bullet}^2 - n\bar{X}_{\bullet\bullet}^2\right) = \Sigma J_i E\left(\bar{X}_{i\bullet}^2\right) - nE\left(\bar{X}_{\bullet\bullet}^2\right)$$

$$= \Sigma J_i \left[Var\left(\bar{X}_{i\bullet}\right) + \left(E\left(\bar{X}_{i\bullet}\right)\right)^2\right] - n\left[Var\left(\bar{X}_{\bullet\bullet}\right) + \left(E\left(\bar{X}_{\bullet\bullet}\right)\right)^2\right]$$

$$= \Sigma J_i \left[\frac{\sigma^2}{J_i} + \mu_i^2\right] - n\left[\frac{\sigma^2}{n} + \frac{(\Sigma J_i \mu_i)^2}{n}\right] = (I-1)\sigma^2 + \Sigma J_i(\mu + \alpha_i)^2 - [\Sigma J_i(\mu + \alpha_i)]^2$$

$$= (I-1)\sigma^2 + \Sigma J_i \mu^2 + 2\mu \Sigma J_i \alpha_i + \Sigma J_i \alpha_i^2 - [\mu \Sigma J_i]^2$$

$= (I-1)\sigma^2 + \Sigma J_i \alpha_i^2$, from which E(MSTr) is obtained through division by $(I-1)$.

10.42 (new edition: Page 439, old edition: Page 431)

a). $\mu_i =$ true average CFF for the three iris colors. Then the hypotheses are $H_0: \mu_1 = \mu_2 = \mu_3$ vs. H_a: at least two μ_i's differ. SST = 13,659.67 – 13,598.36 = 61.31, $SSTR = \left(\frac{(204.7)^2}{8} + \frac{(134.6)^2}{5} + \frac{(169.0)^2}{6}\right) - 13{,}598.36 = 23.00$ The ANOVA table follows:

Source	Df	SS	MS	F
Treatments	2	23.00	11.50	4.803
Error	16	38.31	2.39	
Total	18	61.31		

Because $F_{.05,2,16} = 3.63 < 4.803 < F_{.01,2,16} = 6.23$, .01 < p-value < .05, so we reject H_o. There are differences in CFF based on iris color.

b). $Q_{.05,3,16} = 3.65$ and $W_{ij} = 3.65 \cdot \sqrt{\frac{2.39}{2}\left(\frac{1}{J_i} + \frac{1}{J_j}\right)}$, so the Modified Tukey intervals are:

Pair	$(\bar{x}_{i\bullet} - \bar{x}_{j\bullet}) \pm W_{ij}$
1,2	-1.33 ± 2.27
1,3	-2.58 ± 2.15 *
2,3	-1.25 ± 2.42

Brown	Green	Blue
25.59	26.92	28.17

The CFF is only significantly different for Brown and Blue iris color.

10.46 (new edition: Page 440, old edition: Page 432)

The ordered residuals are –6.67, -5.67, -4, -2.67, -1, -1, 0, 0, 0, .33, .33, .33, 1, 1, 2.33, 4, 5.33, 6.33. The corresponding z percentiles are –1.91, -1.38, -1.09, -.86, -.67, -.51, -.36, -.21, -.07, .07, .21, .36, .51, .67, .86, 1.09, 1.38, and 1.91. The resulting plot of the respective pairs (the Normal Probability Plot) is reasonably straight, and thus there is no reason to doubt the normality assumption.

Chapter 11: # 6, 8, 10, 13, 14, 17 and 18

Chapter 11. Multifactor Analysis of Variance

11.6 (new edition: Page 454, old edition: Page 446)

a). $MSA = \frac{11.7}{2} = 5.85$, $MSE = \frac{25.6}{8} = 3.20$, $f = \frac{5.85}{3.20} = 1.83$, which is not significant at level .05.

b). Otherwise extraneous variation associated with houses would tend to interfere with our ability to assess assessor effects. If there really was a difference between assessors, house variation might have hidden such a difference. Alternatively, an observed difference between assessors might have been due just to variation among houses and the manner in which assessors were allocated to homes.

11.8 (new edition: Page 455, old edition: Page 447)

a). $x_{1\bullet} = 4.34$, $x_{2\bullet} = 4.43$, $x_{3\bullet} = 8.53$, $x_{\bullet\bullet} = 17.30$, $SST = 3.8217$,

$SSTr = 1.1458$, $SSBl = \frac{32.8906}{3} - 9.9763 = .9872$, $SSE = 1.6887$,

$MSTr = .5729$, $MSE = .0938$, f = 6.1. Since $6.1 \geq F_{.05,2,18} = 3.55$, H_{oA} is rejected; there appears to be a difference between anesthetics.

b). $Q_{.05,3,18} = 3.61$, w = .35. $\bar{x}_{1\bullet} = .434$, $\bar{x}_{2\bullet} = .443$, $\bar{x}_{3\bullet} = .853$, so both anesthetic 1 and anesthetic 2 appear to be different from anesthetic 3 but not from one another.

11.10 (new edition: Page 455, old edition: Page 447)

Source	Df	SS	MS	f
Method	2	23.23	11.61	8.69
Batch	9	86.79	9.64	7.22
Error	18	24.04	1.34	
Total	29	134.07		

$F_{.01,2,18} = 6.01 < 8.69 < F_{.001,2,18} = 10.39$, so .001 < p-value < .01, which is significant. At least two of the curing methods produce differing average compressive strengths. (With p-value < .001, there are differences between batches as well.)

$Q_{.05,3,18} = 3.61$; $w = (3.61)\sqrt{\frac{1.34}{10}} = 1.32$

Method A	Method B	Method C
29.49	31.31	31.40

Methods B and C produce strengths that are not significantly different, but Method A produces strengths that are different (less) than those of both B and C.

11.13 (new edition: Page 445 - 446, old edition: Page 447)

a). With $Y_{ij} = X_{ij} + d$, $\bar{Y}_{i\bullet} = \bar{X}_{i\bullet} + d$, $\bar{Y}_{\bullet j} = \bar{X}_{\bullet j} + d$, $\bar{Y}_{\bullet\bullet} = \bar{X}_{\bullet\bullet} + d$, so all quantities inside the parentheses in (11.5) remain unchanged when the Y quantities are substituted for the corresponding X's (e.g., $\bar{Y}_{i\bullet} - \bar{Y}_{\bullet\bullet} = \bar{X}_{i\bullet} - \bar{X}_{\bullet\bullet}$, etc.).

b). With $Y_{ij} = cX_{ij}$, each sum of squares for Y is the corresponding SS for X multiplied by c^2. However, when F ratios are formed the c^2 factors cancel, so all F ratios computed from Y are identical to those computed from X. If $Y_{ij} = cX_{ij} + d$, the conclusions reached from using the Y's will be identical to those reached using the X's.

11.14 (new edition: Page 456, old edition: Page 448)

$$E\left(\overline{X}_{i\bullet} - \overline{X}_{\bullet\bullet}\right) = E\left(\overline{X}_{i\bullet}\right) - E\left(\overline{X}_{\bullet\bullet}\right) = \frac{1}{J} E\left(\sum_j X_{ij}\right) - \frac{1}{IJ} E\left(\sum_i \sum_j X_{ij}\right)$$
$$= \frac{1}{J} \sum_j \left(\mu + \alpha_i + \beta_j\right) - \frac{1}{IJ} \sum_i \sum_j \left(\mu + \alpha_i + \beta_j\right)$$
$$= \mu + \alpha_i + \frac{1}{J} \sum_j \beta_j - \mu - \frac{1}{I} \sum_i \alpha_i - \frac{1}{J} \sum_j \beta_j = \alpha_i$$, as desired.

11.17 (new edition: Page 463, old edition: Page 456)

a).

Source	Df	SS	MS	f	$F_{.05}$
Sand	2	705	352.5	3.76	4.26
Fiber	2	1,278	639.0	6.82*	4.26
Sand&Fiber	4	279	69.75	0.74	3.63
Error	9	843	93.67		
Total	17	3,105			

There appears to be an effect due to carbon fiber addition.

b).

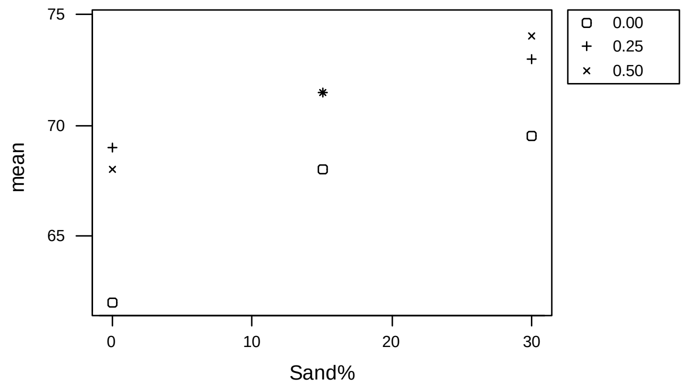
Source	Df	SS	MS	f	$F_{.05}$
Sand	2	106.78	53.39	6.54*	4.26
Fiber	2	87.11	43.56	5.33*	4.26
Sand&Fiber	4	8.89	2.22	.27	3.63
Error	9	73.50	8.17		
Total	17	276.28			

There appears to be an effect due to both sand and carbon fiber addition to casting hardness.

c).

Sand%	Fiber%	$\bar{x}$
0	0	62
15	0	68
30	0	69.5
0	.25	69
15	.25	71.5
30	.25	73
0	.50	68
15	.50	71.5
30	.50	74

The plot below indicates some effect due to sand and fiber addition with no significant interaction. This agrees with the statistical analysis in part **b).**



11.18 (new edition: Page 464, old edition: Page 456)

Source	Df	SS	MS	f	$F_{.05}$	$F_{.01}$
Formulation	1	2,253.44	2,253.44	376.2**	4.75	9.33
Speed	2	230.81	115.41	19.27**	3.89	6.93
Formulation & Speed	2	18.58	9.29	1.55	3.89	6.93
Error	12	71.87	5.99			
Total	17	2,574.7				

a). There appears to be no interaction between the two factors.

b). Both formulation and speed appear to have a highly statistically significant effect on yield.

c). Let formulation = Factor A and speed = Factor B. The estimations are:

For Factor A: $\mu_{1\bullet} = 187.03$ $\mu_{2\bullet} = 164.66$

For Factor B: $\mu_{\bullet 1} = 177.83$ $\mu_{\bullet 2} = 170.82$ $\mu_{\bullet 3} = 178.88$

For Interaction: $\mu_{11} = 189.47$ $\mu_{12} = 180.6$ $\mu_{13} = 191.03$

$\mu_{21} = 166.2$ $\mu_{22} = 161.03$ $\mu_{33} = 166.73$

Overall mean: $\mu = 175.84$

$\alpha_i = \mu_{i\bullet} - \mu$: $\alpha_1 = 11.19$ $\alpha_2 = -11.18$

$\beta_j = \mu_{\bullet j} - \mu$: $\beta_1 = 1.99$ $\beta_2 = -5.02$ $\beta_3 = 3.04$

$\gamma_{ij} = \mu_{ij} - (\mu + \alpha_i + \beta_j)$:

$\gamma_{11} = .45$ $\gamma_{12} = -1.41$ $\gamma_{13} = .96$

$\gamma_{21} = -.45$ $\gamma_{22} = 1.39$ $\gamma_{23} = -.97$

d).

Observed	Fitted	Residual	Observed	Fitted	Residual
189.7	189.47	0.23	161.7	161.03	0.67
188.6	189.47	-0.87	159.8	161.03	-1.23
190.1	189.47	0.63	161.6	161.03	0.57

165.1	166.2	-1.1	189.0	191.03	-2.03
165.9	166.2	-0.3	193.0	191.03	1.97
167.6	166.2	1.4	191.1	191.03	0.07
185.1	180.6	4.5	163.3	166.73	-3.43
179.4	180.6	-1.2	166.6	166.73	-0.13
177.3	180.6	-3.3	170.3	166.73	3.57

e).

From the graphs below, we accept normality and homoscedasticity of residuals.