

## THE GEOMETRIC MEAN PRINCIPLE REVISITED

### A Reply to a 'Reply'

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#### 1. Introduction

Latané's 'Reply' (sic) to my paper,<sup>1</sup> I am afraid, generates more heat than light – as the saying goes. I welcome this opportunity to clarify the issues on which Latané and I are at odds, as well as those on which we now seem to be in agreement.

In my paper, I took issue with two separate arguments put forth by those who claim that for the long-run investor, the policy which maximizes the geometric mean rate of return of his portfolio ('the *G* policy' for short) is best. The first I consider a fallacy. The second is Latané's proposed subgoal for the investor, whose undesirable properties I try to point out.

#### 2. The fallacy

The following are false statements, sometimes implied and sometimes explicitly made by various proponents of the *G* policy:

- (1.F) The *G* policy maximizes the long-run (geometric) mean rate of return (the growth rate).
- (2.F) The *G* policy maximizes terminal wealth. And, consequently,
- (3.F) The *G* policy is optimal for any investor, irrespective of his preferences (utility function), as long as he prefers more wealth to less.

The corresponding correct statements are:

- (1.T) The *G* policy maximizes the mathematical expectation of the long-run growth rate.

<sup>1</sup>Latané (1978) and Ophir (1978).

- (2.T) In the long-run, it is almost certain that by choosing the  $G$  policy, the investor will end up with more wealth than by choosing any other policy.
- (3.T) The optimal policy of the long-run investor depends upon his preferences (utility function).

With regard to (3.T), it is useful to point out that for a person having a logarithmic utility function, the  $G$  policy is optimal in both the short-run and the long-run, whereas a risk-neutral investor should choose the portfolio maximizing the *arithmetic* mean rate of return, in the long-run just as in the short-run. (These polar cases are worth noting irrespective of whether persons of either type actually exist or not.)

I am gratified to learn that Latané now apparently agrees to at least (1.T) and (2.T) of the above statements.

### 3. Latané's subgoal

The advocacy of the  $G$  policy in Latané (1959) is indeed not based on (1.F) or (2.F), but on postulating a certain subgoal for the investor:<sup>2</sup> 'The subgoal proposed here is the choice of the portfolio that has a greater probability ( $P'$ ) of being as valuable or more valuable than other significantly different portfolio at the end of  $n$  years,  $n$  being large' (p. 146). Latané faults me because (in what he chooses to call my 'silly' illustration) I apply his subgoal to a single-period example. Yet I refuse to accept the proposition that the pursuit of goals or subgoals, as distinct from policies, should be made dependent on particular circumstances.<sup>3</sup>

If a certain subgoal is desirable for some  $n$ , ' $n$  being large', it should also be worth pursuing for the case of  $n=1$ . Indeed, consider the tabulation of the probability distributions of outcomes in my paper (p. 105). There they appear as the result of a long-run process ( $n=10$  and  $n=20$ , respectively). Should these prospects be evaluated on the basis of a different (sub)goal than prospects resulting in identical distributions of outcomes in a single-period decision?

A switching of objectives is stated quite explicitly in the last paragraph of Latané (1978): My illustrative gamble, he says, should be accepted as a one-time proposition on the basis of 'almost any utility consideration', whereas the multiple gamble would be accepted on the basis of the high probability

<sup>2</sup>I am sorry if my paper inadvertently gave a different impression. However, if I wanted to be polemic, I could point to the text book by Latané, Tuttle and Jones (1975), whose admittedly loose formulations certainly create the impression that the  $G$  policy maximizes wealth in the long-run (pp. 564–565).

<sup>3</sup>Latané misquotes himself when he states that the criterion (instead of the subgoal) is only applicable for repeated choices.

of an (enormous) gain; the latter basis is akin to, although not identical with, the subgoal. This approach is particularly bothersome if we recall that Latané (1959) expressly proposed his subgoal as an alternative to that of maximizing expected utility. Moreover, the two are incompatible; the adoption of the objective of maximizing the probability of being better off constitutes a rejection of the von Neumann–Morgenstern concept of utility. To see this, just consider the well-known experiment by which a person's utility function is constructed:<sup>4</sup> We choose two arbitrary wealth levels, say \$0 and \$100, and assign to them the utility levels of 0 and 1, respectively. For amounts \$*a* between \$0 and \$100, we ask for the probability *p* making the person indifferent between \$*a* with certainty and a gamble for \$100. Now, for any amount \$*a*, Latané will prefer the certainty if  $p < 1/2$ , and the gamble if  $p > 1/2$ . Hence, the utility level of any amount between \$0 and \$100 is exactly 0.5. By a similar argument, it can be established that any amount greater than \$100 has a utility level of exactly 2. But the initial choice of \$100 was arbitrary. By repeating the experiment with different amounts, we get new utility functions which are inconsistent with each other.

Thus, a person accepting Latané's subgoal has to forego not only expected utility but the concept of utility itself.

<sup>4</sup>See, e.g., Luce and Raiffa (1957, pp. 21–22).

## References

- Latané, H.A., 1959, Criteria for choice among risky ventures, *Journal of Political Economy* 67, 144–155.
- Latané, H.A., D.L. Tuttle and C.P. Jones, 1975, *Security analysis and portfolio management*, 2nd ed. (Ronald Press, New York).
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