Statistics 434 Homework No. 1

Experience with S functions and Simulation

- Write a function getrhohat() which creates a realization of $\hat{\rho}$, the sample autocorrelation for the series $\{y_t\}$ which is a simulated realization of length $T = 100$ from a stationary AR model with $\rho = 0.3$ and $\sigma^2 = 1$.
  1. Note that we have already developed in class all the steps that are needed to generate $\{y_t\}$. Here you can use the fact that we know the distribution of y_1 that will make the series stationary.
  2. Your task is to put the generation of $\{y_t\}$ and the computation of a realization of $\hat{\rho}$ into a neat package.
- Use your function getrhohat() and a "for loop" to create a vector of length 1000 that contains 1000 independent realizations of $\hat{\rho}$. Call the vector of these realizations rhosample. Note: You will probably want to create an "empty version" of rhosample by assigning rhosample equal to rep(0,1000). Your "for loop" will then call getrhohat() a thousand times to replace the zeros in your vector with honest independent realizations of $\hat{\rho}$.
- Use rhosample to examine the validity of the assertion that $\hat{\rho}$ is approximately *normally distributed* with mean $\rho = 0.3$ and variance $1/T$.
- You can address the issue of normality using qualitative graphical tools such as the histogram or the normal quantile plot. We will discuss formal tests of normality later in the course.
- What to hand in? Give a one page summary of what you discovered and experienced. You can attach to this your code, qqnorm plot, and histogram. Do not print out 1000 numbers!

Some Perspective

In this exercise we use simulation to help us to achieve a deeper understanding of the distribution of an estimator of a parameter. Specifically, $\hat{\rho}$ estimates the ρ of an AR model, and we want to know how well $\hat{\rho}$ does the job.

You are encouraged to think about the logic of this experimental approach to understanding the *random variable* $\hat{\rho}$. This is a good time to underline the distinction between a *parameter*, like ρ and a *statistic* (or *estimator*) like $\hat{\rho}$ which is a random variable.

Here we know that ρ is exactly 0.3, but $\hat{\rho}$ takes on changing values that depend on the luck of the draw. To understand $\hat{\rho}$ it is necessary to understand its distribution, and simulation provides us with a very convenient tool.