Statistics 434 Homework No. 2: An Experiment with Ljung-Box

First recall the Ljung-Box Statistic on k lags:

$$LB = T(T+2)\sum_{j=1}^{k} \frac{\hat{\rho}_j{}^2}{T-j}$$

- Write an S-Plus function that computes a vector that holds the first 25 sample autocorrelations of a time series. Hint: The lion's share of the work is already done by acf(); you just need to find a way to extract the information from what acf() returns.
- Write a function that computes the kth Box-Ljung statistic for a time series.
- Under the null hypothesis that the series is Gaussian white noise, the distribution of the Ljung-Box statistic *should* approximately equal that of the $\chi^2(k)$ distribution. Your main task is to design and run a simulation experiment to check this.
  1. You might want to begin by generating 1000 values of the Ljung-Box statistic under the null hypothesis. Here you might want to follow the pattern of HW1; that is, write a function that (1) generates the Gaussian white noise series and (2) calculates and returns the Ljung-Box statistic. You can then use a "for loop" to fill up a 1000 vector by calling on your function 1000 times.
  2. You could then examine this sample to see if it is compatible with the $\chi^2(k)$ distribution. An appropriate qqplot seems in order. Naturally you want to compare your Ljung-Box simulated values with a Chi-squared with the k degrees of freedom.
  3. You will need to make some choices in your experiment. In particular, you will want to study more than one value of k. Which k's make sense to you? How can you present your results in a nice way?
- There are many ways to present your experiment. As a baseline, you should consider using (1) a one page executive summary and (2) just one or two *well-designed* graphs. Here the qqplot will surely come into consideration, but for a really great picture you will also want to think about ways to use labels, titles, and par(mfrow=c(2,2)). Presentation is an important part of a statistical analysis, and each of our HWs will ramp up the quality of presentation that is required.

Perspective

This homework covers considerable ground. You will have added several new S-tools to your kit, and you will have engaged one of the most fundamental notions of traditional time series analysis — the autocorrelation function. You also now have a way to test one of the most persistent alternative hypotheses: *the hypothesis of white noise.*