Statistics 434 Homework No. 5: Kelly Betting on AR(1)

This homework is more open-ended than those we have done before. It provides plenty of room for you to exercise your good sense and to show that you can conduct a wise exploration of the ideas that have been developed in class. You will need to make many choices without being given too many hints. This is part of the experience.

- Suppose that an asset has weekly returns that satisfy the AR(1) model with $\mu = 0.002$, $\phi = 0.2$ and $\sigma = 0.04$. This is a crude, but sensible, approximation for the US Total market during a certain 20 year period. Build an S-plus function that returns the wealth process for a investor who makes investments each week according to the modified full Kelly criterion where (1) he cannot go short — so does not bet when he has negative expectation and (2) just bets his full wealth when the Kelly factor is larger than one — so no margin purchases. Look at the wealth time series for some number of independent replications of this simulation with T=1000 (about 20 years). Comment on your results.

- Now generalize your function so that you can explore other sets of assumptions about the AR(1) model parameters that you now choose. Explore the sensitivity of the wealth process to changes in model parameters and comment on your results.

- Modify your function to permit short sales up to the size of your capital and leverage up to the size of your capital. Assume at this stage that there is no cost to borrow shares and no cost to leveraging. Explore and comment.

- Improve your function so that it accommodates fractional Kelly bet sizes. Explore and comment.

- Finally, **as an option**, explore the impact of margin costs on Kelly guided bet sizes in the context of an AR(1) model focusing on the long-only situation. Using a Fermi-type calculation and assuming that you pay margin on your whole investment, how much does this reduce your Kelly bet size? Based on this, make a practical suggestion about the approximate Full Kelly bet size where you just pay margin on the funds that you actually have to borrow. Your answer is a seat-of-the pants back-of-the-envelope thing — but it should be sensible if you take up the task.

This is intended to be a challenging assignment with lots of opportunity for individual choices, both in what is actually done and in how the results are presented. Still, please limit yourself to two pages for your executive summary. Please also make sure that your graphs are done as informatively as possible (say with help from multi-frame plots).