

**SIR GILBERT WALKER AND A CONNECTION  
BETWEEN EL NIÑO AND STATISTICS**

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**REFERENCE: KATZ, R. W. (2002). *STATISTICAL  
SCIENCE*, V. 17, PP. 97–112.**

## OPENING QUOTE

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**SIR GILBERT WALKER (1925):**

*“IT IS A NATURAL SUPPOSITION THAT THERE SHOULD BE IN WEATHER FREE OSCILLATIONS WITH FIXED NATURAL PERIODS, AND THAT THESE OSCILLATIONS SHOULD PERSIST EXCEPT WHEN SOME EXTERNAL DISTURBANCE PRODUCES DISCONTINUOUS CHANGES IN PHASE OR AMPLITUDE.”*

# **OUTLINE**

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**(1) BACKGROUND**

**(2) WORLD WEATHER / CENTERS OF ACTION**

**(3) YULE–WALKER EQUATIONS**

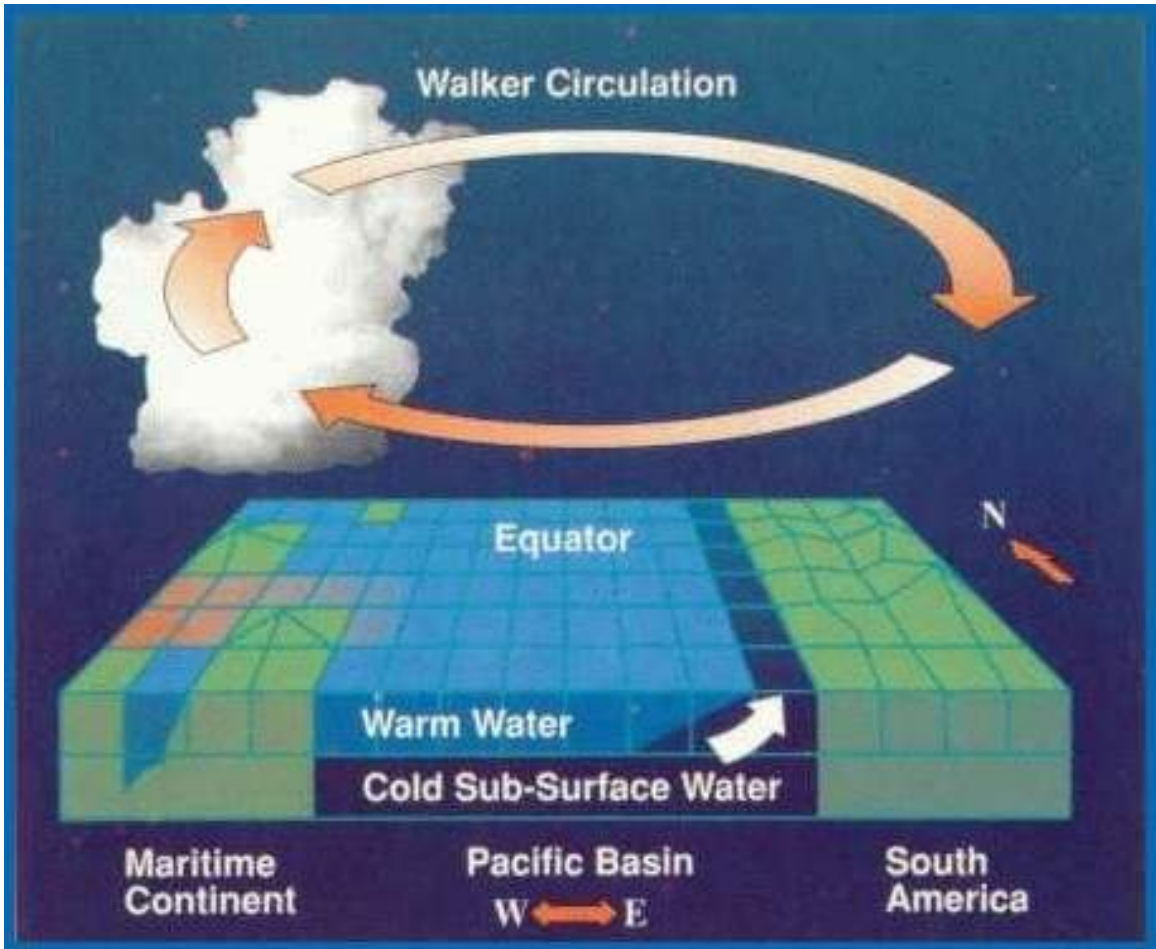
**(4) REACTION TO WALKER'S WORK**

**(5) PRESENT SITUATION**

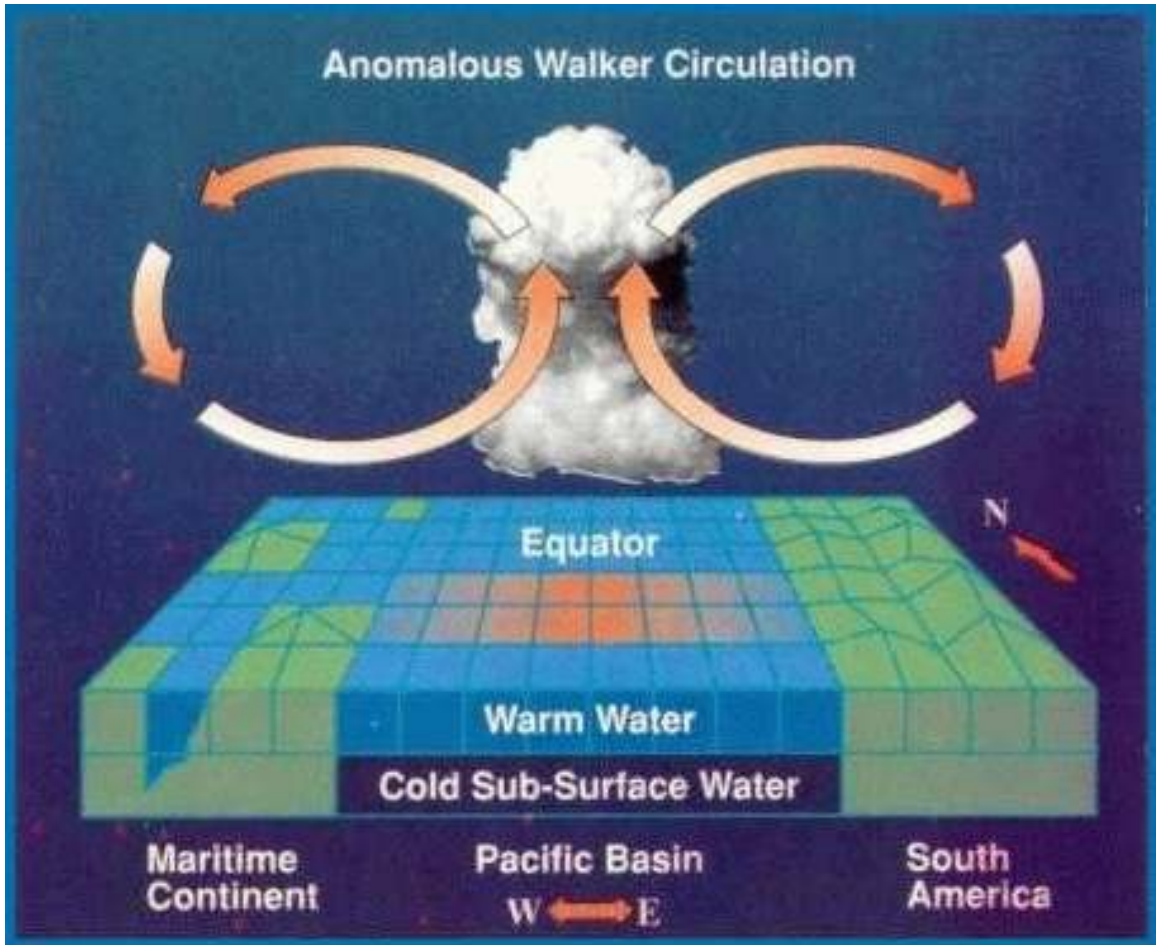
## **(1) BACKGROUND**

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- **EL NIÑO / SOUTHERN OSCILLATION (ENSO)**
  - **1997–1998 EL NIÑO EVENT**
  - **EL NIÑO (“CHRIST CHILD”)**
  - **TELECONNECTIONS**
  - **SOUTHERN OSCILLATION (SO)**
  - **CENTERS OF ACTION / PRESSURE SEESAW**
  - **“WALKER CIRCULATION” (BJERKNES, 1969)**



**SOURCE: NASA ENSO PRIMER**



**SOURCE: NASA ENSO PRIMER**

- **QUASIPERIODIC BEHAVIOR**

- **$p$ TH-ORDER AUTOREGRESSIVE PROCESS [AR( $p$ )]**

- **“YULE–WALKER EQUATIONS” [KENDALL, 1949]**

**YULE (1927):**

**AR(2) MODEL FOR SUNSPOTS**

**WALKER (1931):**

**AR(4) MODEL FOR DARWIN PRESSURE**

**USE TODAY:**

**PARTIAL AUTOCORRELATIONS (E. G., S-PLUS)**

**MULTIPLE AR MODELS (PARAMETER**

**ESTIMATION)**

## **(2) WORLD WEATHER / CENTERS OF ACTION**

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### **• TRAINING & CAREER OF WALKER**

- 1868**            **BORN (LANCASHIRE, ENGLAND)**
- 1883**            **LINGUISTIC BLUNDER**
- 1886–1890**      **MATHEMATICS, TRINITY COLLEGE**
- 1891–1893**      **WINTERS IN SWITZERLAND**
- 1892**            **PUBLISHED FIRST PAPER**
- 1895–1903**      **LECTURER, TRINITY COLLEGE**
- 1903–1924**      **INDIA METEOROLOGICAL DEPT.**
- 1924–1934**      **IMPERIAL COLLEGE, UNIV. LONDON**
- 1934**            **RETIREMENT**
- 1950**            **LAST PAPER (*BIOMETRIKA*)**
- 1958**            **DIED (SURREY, ENGLAND)**



- **STATISTICAL RESEARCH OF WALKER**

- **CORRELATION & REGRESSION**

**CENTERS OF ACTION:**

**SOUTHERN OSCILLATION (SO)**

**NORTH ATLANTIC OSCILLATION (NAO)**

**NORTH PACIFIC OSCILLATION (NPO)**

- **MULTIPLE COMPARISONS**

**WALKER TEST (CORRELATIONS / PERIODOGRAM )**

- **HARMONIC ANALYSIS**

**“PERIOD-HUNTING” (CAUTIONED AGAINST)**

### (3) YULE-WALKER EQUATIONS

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- AR PROCESS [ZERO-MEAN AR( $p$ ) PROCESS  $X_t$ ]

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + a_t$$

$\phi_k$   $k$ TH-ORDER AUTOREGRESSION COEFFICIENT

$a_t$  ZERO MEAN, UNCORRELATED PROCESS

#### – YULE-WALKER EQUATIONS

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p} \quad k \neq 0$$

$$\sigma_a^2 = (1 - \phi_1 \rho_1 - \phi_2 \rho_2 - \cdots - \phi_p \rho_p) \sigma^2$$

$$\rho_k = \text{corr}(X_t, X_{t-k}) \quad \sigma^2 = \text{var}(X_t) \quad \sigma_a^2 = \text{var}(a_t)$$

## **– QUASIPERIODS**

**AR( $p$ ) PROCESS,  $p \geq 2$ :**

**CAPABLE OF PRODUCING QUASIPERIODIC  
BEHAVIOR**

**(i) CORRELATION QUASIPERIOD**

**DAMPED OSCILLATION IN AUTOCORRELATION  
FUNCTION**

**(ii) SPECTRAL QUASIPERIOD**

**PEAK (BROAD BAND) IN SPECTRUM**

- **YULE'S MODEL FOR SUNSPOT NUMBERS**

- **SINE WAVE RANDOMLY SHIFTED IN PHASE & AMPLITUDE**

**PENDULUM ANALOGY: “UNFORTUNATELY BOYS GET INTO THE ROOM AND START PELTING THE PENDULUM WITH PEAS, SOMETIMES FROM ONE SIDE AND SOMETIMES FROM THE OTHER” (YULE, 1927)**

- **AR(2) PROCESS**

**FIT BY LEAST SQUARES TO ANNUAL SUNSPOT TIME SERIES, 1749–1924 (CORRELATION QUASIPERIOD  $\approx$  11 YRS)**

- **WALKER'S MODEL FOR DARWIN PRESSURE**

**SEASONAL INDEX OF SOUTHERN OSCILLATION**

– **DERIVATION OF YULE–WALKER EQUATIONS**

**EXTENDED YULE'S AR(2) MODEL TO**

**GENERAL AR( $p$ ) PROCESS,  $p \geq 1$**

– **ANALYSIS OF DARWIN PRESSURE**

**DECIDED ON AR(4) MODEL**

**DID NOT USE LEAST SQUARES**

**CORRELATION QUASIPERIOD  $\approx$  3 YRS**

## **– REANALYSIS OF DARWIN PRESSURE**

### **MODIFICATIONS:**

**CORRECT FOR INSTRUMENTAL BIAS**

**REMOVE ANNUAL CYCLE IN STANDARD**

**DEVIATION**

**PARAMETER ESTIMATION BY YULE–WALKER**

**EQUATIONS**

### **RESULTS:**

**DAMPED OSCILLATION IN AUTOCORRELATION**

**FUNCTION**

**AR(4) MODEL GIVES SPECTRAL QUASIPERIOD  $\approx$**

**3 – 3 ½ YRS**

## **(4) REACTION TO WALKER'S WORK**

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- **REACTION IN STATISTICS**

**WOLD (1938): BOOK MARKED EMERGENCE OF MODERN TIME SERIES ANALYSIS (BUT CITED ONLY TWO APPLICATIONS OF AR PROCESSES)**

**(i) YULE (1927) SUNSPOTS APPLICATION**

**[WOLD: PRAISED AR(2) MODEL]**

**(ii) WALKER (1931) PRESSURE APPLICATION**

**[WOLD: SKEPTICAL ABOUT AR(4) MODEL;**

**THOUGHT AR(1) MODEL ADEQUATE (I. E., NO**

**QUASIPERIODIC BEHAVIOR)]**

- **REACTION IN METEOROLOGY**

- **GENERAL SKEPTICISM ABOUT STATISTICS**

- **PHYSICAL EXPLANATION (WALKER SOUGHT, BUT APPEALS UNHEEDED)**

**SHEPPARD (1959):**

***“WALKER’S HOPE WAS PRESUMABLY NOT ONLY TO UNEARTH RELATIONS USEFUL FOR FORECASTING BUT TO DISCOVER SUFFICIENT AND SUFFICIENTLY IMPORTANT RELATIONS TO PROVIDE A PRODUCTIVE STARTING POINT FOR A THEORY OF WORLD WEATHER. IT HARDLY APPEARS TO BE WORKING OUT LIKE THAT.”***



## **(5) PRESENT SITUATION**

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- **QUASIPERIODIC BEHAVIOR**

- **CONSENSUS THAT ENSO PHENOMENON**

**EXHIBITS QUASIPERIODIC BEHAVIOR**

- **NONLINEAR DYNAMICS**

- **QUASIPERIODIC FEATURE OF ENSO VIEWED**

**AS FUNDAMENTAL**

- **SOURCE REMAINS UNCLEAR**

## **CLOSING QUOTE**

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**SIR GILBERT WALKER (1927):**

**“THERE IS, TO-DAY, ALWAYS A RISK THAT SPECIALISTS IN TWO SUBJECTS, USING LANGUAGES FULL OF WORDS THAT ARE UNINTELLIGIBLE WITHOUT STUDY, WILL GROW UP NOT ONLY, WITHOUT KNOWLEDGE OF EACH OTHER’S WORK, BUT ALSO WILL IGNORE THE PROBLEMS WHICH REQUIRE MUTUAL ASSISTANCE.”**