A State-Space Forecasting Approach to Optimal Intertemporal Cross-Hedging

Tomislav Vukina and James L. Anderson

Cross-commodity hedging between fishmeal and soybean meal is investigated. The approach uses successively updated out-of-sample forecasts to approximate subjective price expectations, and forecast error variance-covariance matrices to measure risk. Forecasts are generated by state-space models of vector-valued time series. In a stationary environment, uncertainty reduces to the difference between the historical autocovariance of the random process and the variance-covariance of out-of-sample forecasts. Results indicate that weakly risk-averse agents can increase average marketing returns within acceptable risk levels by combining information from price forecasting models with an appropriate hedging strategy.

Key words: dynamic cross-hedging, fishmeal, state-space forecasting, uncertainty.

In this paper we investigate cross-commodity hedging possibilities between a processed renewable natural resource (fishmeal) and a processed agricultural commodity (soybean meal). Our objectives are to design a dynamic hedging model based on state-space time-series price forecasts, and to compare model performance with cash marketing and with routine and static hedging. An optimal strategy is developed for a hypothetical fishmeal producer (dealer), with futures positions established in soybean meal contracts.

Rooted in the method of Anderson and Danthine, our intertemporal hedging model allows a futures position to be revised within the cash position holding period. Hedging behavior is cast in a portfolio selection framework in which cash position is exogenous, so that the problem becomes one of determining the futures position time path maximizing expected utility of the agent's monetary wealth at the end of the trading period.

The approach uses successively updated out-of-sample price forecasts to approximate subjective price expectations, and the variance-covariance matrices of forecast errors to measure risk. Forecasts are generated by the state-space technique of modeling vector-valued time series (Aoki). The model is repeatedly solved each period after updated information becomes available. The result is a set of time-varying hedge ratios. We show that uncertainty, defined as the variance-covariance of out-of-sample forecast error (mean squared out-of-sample forecast error), is in fact the difference between the unconditional (historical) autocovariance of the process and the variance-covariance of the out-of-sample forecasts.

Multiperiod Hedging Model

A commodity trader starts with initial wealth W_0, which he invests entirely in a commodity for storage and later resale. Assuming a futures market in this or a related commodity, the investor can hedge his cash position by selling futures contracts maturing at or after the end of the hedging period. We assume unlimited borrowing and lending at a non-stochastic interest rate.

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Consider a three-period example. In period 0, after the cash position has been established, the trader decides on the initial futures position h_0. The futures position is adaptive in the sense that quantity hedged in period 1 generally differs from the quantity hedged in period 0. At the end of the second period, the dealer closes all outstanding futures positions, sells the cash commodity, and collects the proceeds. Assuming an increasing, strictly concave, twice differentiable Von Neumann-Morgenstern utility function U with terminal wealth W_2 as the sole argument, the first-step optimization problem can be formulated as

$$(1) \qquad \max_{h_0, h_1} J_0 = \mathrm{E}[U(W_2)|Z_0]$$

where $W_2 = (1 + r)^2(-\bar{p}_0 q_0^*) + (1 + r)(\bar{f}_0 - f_1)h_0 + (f_1 - f_2)h_1 + p_2 q_0^*.$

Variable $\bar{p}_0$ is initial price at which the exogenously determined cash commodity q_0^* is purchased; p_2 is stochastic price at which the cash commodity (risky asset) must be sold; $\bar{f}_0$ is futures price available at time a decision is made; f_1 and f_2 are stochastic futures prices in the respective periods; r is the one-period risk-free interest rate; h_0 and h_1 are decision variables denoting quantity of futures sold (purchased if negative); and Z_0 is information available (history of cash and futures prices, including data points p_0 and f_0) at beginning of period 0 when the first decision is made. Terminal monetary wealth W_2 reflects the fact that the trader's futures account is marked to the market, meaning all profits or losses related to the futures position are realized each period (Cox, Ingersoll, and Ross).

The two first-order conditions for an extremum are

$$(2) \qquad \frac{\partial J_0}{\partial h_0} = \mathrm{E}[U'(W_2)](\bar{f}_0 - \hat{f}_1) - \mathrm{cov}[U'(W_2), f_1] = 0$$

$$\frac{\partial J_0}{\partial h_1} = \mathrm{E}[U'(W_2)](\hat{f}_1 - \hat{f}_2) + \mathrm{cov}[U'(W_2), f_1] - \mathrm{cov}[U'(W_2), f_2] = 0$$

where expectations operator E represents the trader's subjective expectations conditional on information Z_0, $\hat{f}_1 = \mathrm{E}(f_1|Z_0)$, and $\hat{f}_2 = \mathrm{E}(f_2|Z_0)$. Because of strict concavity of U, second-order conditions for a maximum are implicitly satisfied. The first-order conditions can be further analyzed using the result of Rubinstein, who showed that if two random variables, such as W_2 and f_1, are jointly normally distributed, and $U(W_2)$ is a twice-differentiable function, then $\mathrm{cov}[U'(W_2), f_1] = \mathrm{E}[U''(W_2)]\,\mathrm{cov}(W_2, f_1)$. Hence

$$(3) \qquad (\bar{f}_0 - \hat{f}_1) - \frac{\mathrm{E}[U''(W_2)]}{\mathrm{E}[U'(W_2)]}\mathrm{cov}(W_2, f_1) = 0$$

$$(\hat{f}_1 - \hat{f}_2) - \frac{\mathrm{E}[U''(W_2)]}{\mathrm{E}[U'(W_2)]}[\mathrm{cov}(W_2, f_2) - \mathrm{cov}(W_2, f_1)] = 0.$$

Assuming a constant absolute risk aversion (CARA) utility function, and normally distributed terminal wealth, term $-\,\mathrm{E}[U''(W_2)]/\mathrm{E}[U'(W_2)]$ in (3) equals the Pratt-Arrow measure of absolute risk aversion λ. Maximizing $\mathrm{E}[U(W_2)]$ then is equivalent to maximizing the mean-variance objective function $J = \mathrm{E}(W_2) - (\lambda/2)\,\mathrm{var}(W_2)$ (Hey, pp. 46–51). Substituting (1) for W_2, and replacing $-\,\mathrm{E}[U''(W_2)]/\mathrm{E}[U'(W_2)]$ with λ, (3) can be rewritten as the linear system:

$$(4) \qquad \mathbf{Mh} = \boldsymbol{\nu}, \quad \text{where}$$

$$\mathbf{M} = \begin{pmatrix} (1 + r)\mathrm{var}f_1 & -(\mathrm{var}f_1 - \mathrm{cov}f_1f_2) \\ -(1 + r)(\mathrm{var}f_1 - \mathrm{cov}f_1f_2) & (\mathrm{var}f_1 - 2\mathrm{cov}f_1f_2 + \mathrm{var}f_2) \end{pmatrix},$$

$$\mathbf{v} = \begin{pmatrix} \dfrac{\bar{f}_0 - \hat{f}_1}{\lambda} + q_0^*\,\mathrm{cov}p_2f_1 \\ \dfrac{\hat{f}_1 - \hat{f}_2}{\lambda} - q_0^*(\mathrm{cov}p_2f_1 - \mathrm{cov}p_2f_2) \end{pmatrix}, \qquad \mathbf{h} = \begin{pmatrix} h_0 \\ h_1 \end{pmatrix}.$$

The solution of (4) for the optimal values of h_0 and h_1 determines the hedging strategy for the entire planning horizon based on information available in period 0. Optimal choices at points 0 and 1 are mutually dependent since they jointly solve the linear system of first-order conditions for an intertemporal maximum. However, the model allows adjustment of the original hedging policy each time period, after new information becomes available. Therefore, at point 0 an agent effectively takes only the futures position $h_0|Z_0$, and never executes his current estimate of next period's optimal futures position $h_1|Z_0$. The latter serves only as an auxiliary decision variable enabling derivation of the optimal intertemporal solution to $h_0|Z_0$.

Using Cramer's rule, the optimal hedge in period 0 is given by

$$(5) \qquad h_0^*|Z_0 = q_0^* \frac{\sigma_{p_2 f_1}(\sigma_{f_2}^2 - \sigma_{f_1 f_2}) + \sigma_{p_2 f_2}(\sigma_{f_1}^2 - \sigma_{f_1 f_2})}{\sigma_{f_1}^2 \sigma_{f_2}^2 - (\sigma_{f_1 f_2})^2} + \frac{(\bar{f}_0 - \hat{f}_1)(\sigma_{f_1}^2 - 2\sigma_{f_1 f_2} + \sigma_{f_2}^2) + (\hat{f}_1 - \hat{f}_2)(\sigma_{f_1}^2 - \sigma_{f_1 f_2})}{\lambda[\sigma_{f_1}^2 \sigma_{f_2}^2 - (\sigma_{f_1 f_2})^2]}.$$

Given cash position q_0^*, the optimal hedge in period 0 is a function of expected returns (prices) in periods 1 and 2, variances and covariances surrounding price expectations, and the generally unknown risk aversion parameter λ. However, with strong risk aversion (sufficiently large λ), the second term in (5) is inconsequential and the optimal futures position is proportional to the existing cash position. The proportionality factor is composed only of the variance-covariance structure of the stochastic cash and futures prices.

A one-period advance in time presents the trader with new data, and hence the original hedging policy is revised. To determine the optimal futures position for the next period, the decision process is repeated given the cash position and the previous period's optimal futures position from (5). In this step, maximizing utility of terminal wealth is based on information Z_1, which has been updated with new data points p_1 and f_1. The optimization problem can be formulated as

$$(6) \qquad \max_{h_1} J_1 = \mathrm{E}[U(W_2)|Z_1]$$

where $W_2 = (1 + r)^2(-\bar{p}_0 q_0^*) + (1 + r)(\bar{f}_0 - \bar{f}_1)h_0^* + (\bar{f}_1 - f_2)h_1 + p_2 q_0^*$.

The remaining stochastic variables are p_2 and f_2, because with the one-period time advancement, $\bar{f}_1$ has become a known constant. The single first-order condition is

$$(7) \qquad \frac{\partial J_1}{\partial h_1} = (\bar{f}_1 - \tilde{f}_2) - \frac{\mathrm{E}[U''(W_2)]}{\mathrm{E}[U'(W_2)]} \mathrm{cov}(W_2, f_2) = 0$$

where E is the trader's subjective expectation conditional on Z_1, and $\tilde{f}_2 = \mathrm{E}(f_2|Z_1)$. Evaluating the covariance term, (7) can be solved for the new optimal futures position $h_1^*|Z_1$. Given q_0^* and $h_0^*|Z_0$, the optimal hedge in period 1 depends on the expected returns in the final period and on the variance-covariance structure of the stochastic prices, both conditional on Z_1:

$$(8) \qquad h_1^*|Z_1 = q_0^* \frac{\sigma_{p_2 f_2}}{\sigma_{f_2}^2} + \frac{\bar{f}_1 - \tilde{f}_2}{\lambda \sigma_{f_2}^2}.$$

Futures positions $h_0^*|Z_0$ and $h_1^*|Z_1$ form the optimal dynamic hedging policy. In contrast to the static model where the hedge for the entire horizon is based on the initial information Z_0 only, the dynamic model uses new information for each period, and adjust the hedging position accordingly. Static hedge, once initiated, remains the same throughout the hedging horizon.[1]

Expectations and Risk: Concepts and Measurement

The proposed method exploits the dynamic relationship between contemporaneous time series of cash and futures prices, forecasting them simultaneously using the state-space modeling technique. Based on the insights of linear systems theory, a state-space method for modeling vector-valued time series has been proposed by Aoki. Cerchi and Havenner apply the method to stock prices. Dorfmann and Havenner use the state-space technique to model the cyclical pattern in supply and demand of California olives.

The state-space model consists of state and observation equations:

$$(9) \qquad \mathbf{x}_{t+1|t} = \mathbf{A}\mathbf{x}_{t|t-1} + \mathbf{B}\mathbf{e}_t$$
$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_{t|t-1} + \mathbf{e}_t$$

where $\{\mathbf{y}_t\}$, to be modeled, is an m-series, zero-mean, weakly stationary vector-valued Gaussian stochastic process with autocovariance sequence $\{\mathbf{\Gamma}_j\}$. Matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are system parameters, $\mathbf{x}_t$ is a vector of n unobservable states

[1] Given a cash position, the general n-period expression for an optimal static hedge is

$$h^*|Z_0 = \frac{\bar{f}_0 - \hat{f}_n}{\lambda \sigma_{f_n}^2} + q_0^* \frac{\sigma_{p_n f_n}}{\sigma_{f_n}^2}.$$

that are minimal sufficient statistics for the history of process $\{\mathbf{y}_t\}$, and innovations $\{\mathbf{e}_t\}$ are white noise. Subscripts on $\mathbf{x}$ refer to the conditional expectation of $\mathbf{x}$ in the period of the first subscript given the information at the time of the second subscript.

Estimates of system matrices and initial state $\mathbf{x}_1$ enable straightforward generation of in-sample and out-of-sample forecasts. Parameter estimation and forecasting procedures of minimal complexity appear in Havenner and Aoki (see also Vukina 1992). Out-of-sample forecasts are given by

$$(10) \qquad \hat{\mathbf{y}}_{t+k+1} = \mathbf{CA}^k\mathbf{x}_{t+1} + \boldsymbol{\mu}.$$

Because centered data are used, the mean of series μ, initially subtracted from each observation, must be added back to (10) to obtain a forecast of the same order of magnitude as the actual data. If $k = 0$, (10) reduces to a one-step forecast; $k = 1$ generates $\hat{\mathbf{y}}_{t+2}$, and so forth. Since contemporaneous cash and futures prices are modeled together, vector $\hat{\mathbf{y}}_{t+1}$ consists of one-step-ahead out-of-sample forecasts of futures price (f_1) and cash price (p_1); and $\hat{\mathbf{y}}_{t+2}$ consists of the two-step-ahead, out-of-sample forecast of f_2 and p_2.

The individual trader's perceived risk is measured by the variance-covariance of the out-of-sample forecast error (mean squared out-of-sample forecast error). Variance of the one-step-ahead forecast error is defined as

$$(11) \quad \boldsymbol{\Sigma}_{11} = \mathrm{E}[(\mathbf{y}_{t+1} - \hat{\mathbf{y}}_{t+1})(\mathbf{y}_{t+1} - \hat{\mathbf{y}}_{t+1})'].$$

Cross-multiplying terms and substituting (10) for $\hat{\mathbf{y}}_{t+1}$ and (9) for $\mathbf{y}_{t+1}$, (11) becomes

$$(12) \quad \boldsymbol{\Sigma}_{11} = \mathrm{E}(\mathbf{y}_{t+1}\mathbf{y}'_{t+1}) - 2\mathrm{E}(\mathbf{Cx}_{t+1}\mathbf{x}'_{t+1}\mathbf{C}' + \mathbf{e}_{t+1}\mathbf{x}'_{t+1}\mathbf{C}') + \mathbf{C}\mathrm{E}(\mathbf{x}_{t+1}\mathbf{x}'_{t+1})\mathbf{C}'.$$

An important feature of weakly stationary stochastic processes is the fact that covariances of all equidistant lags are identical. Invoking this feature, and observing the definition of state covariance matrix $\mathbf{P} = \mathrm{E}(\mathbf{x}_{t+1|_t}\mathbf{x}_{t+1|_t}')$ and the orthogonality of innovations with states [namely $\mathrm{E}(\mathbf{e}_{t+1}\ \mathbf{x}_{t+1|_t}) = 0$], equation (12) becomes

$$(13) \qquad \boldsymbol{\Sigma}_{11} = \boldsymbol{\Gamma}_0 - \mathbf{CPC}'$$

where $\boldsymbol{\Gamma}_0 = \mathrm{E}(\mathbf{y}_t\mathbf{y}_t') = \mathrm{E}(\mathbf{y}_{t+1}\mathbf{y}'_{t+1})$ represents the zero-lag unconditional data autocovariance matrix. The variance of the one-step-ahead forecast error in (13) is an $(m \times m)$ matrix:

$$(14) \qquad \boldsymbol{\Sigma}_{11} = \begin{bmatrix} \sigma^2_{p_1} & \sigma_{p_1f_1} \\ \sigma_{f_1p_1} & \sigma^2_{f_1} \end{bmatrix}$$

where the diagonal elements represent the variance of a one-step-ahead cash price forecast error and the variance of a one-step-ahead futures price forecast error respectively, and the off-diagonal elements are the corresponding forecast error covariances. Similarly, the covariance of the one-step-ahead and two-step-ahead forecast error is defined as

$$(15) \quad \boldsymbol{\Sigma}_{12} = \mathrm{E}[(\mathbf{y}_{t+1} - \hat{\mathbf{y}}_{t+1})(\mathbf{y}_{t+2} - \hat{\mathbf{y}}_{t+2})'] = \boldsymbol{\Gamma}_1' - \mathbf{CPA'C'} = \begin{pmatrix} \sigma_{p_1p_2} & \sigma_{p_1f_2} \\ \sigma_{f_1p_2} & \sigma_{f_1f_2} \end{pmatrix}.$$

where $\boldsymbol{\Gamma}_1' = \mathrm{E}(\mathbf{y}_t\mathbf{y}'_{t+1}) = \mathrm{E}(\mathbf{y}_{t+1}\ \mathbf{y}'_{t+2})$ is the transpose of the one-lag data autocovariance matrix. The transpose of (15) gives the covariance of the two-step-ahead and one-step-ahead forecast error $(\boldsymbol{\Sigma}_{21})$. Variance-covariance matrices of n-step-ahead forecast errors are calculated similarly.

Results show that uncertainty, defined as the variance-covariance of the out-of-sample forecast error (mean squared out-of-sample forecast error), is in fact a difference between the historical (sample) estimate of the lag j autocovariance Γ_j and the variance-covariance of the out-of-sample forecasts $\mathrm{E}(\hat{\mathbf{y}}\hat{\mathbf{y}}')$.[2] The further away the out-of-sample forecasts are from the present, the larger is the mean square forecast error, and therefore the lower is the forecast reliability. Generating n-step-ahead forecasts would gradually bring the variance-covariance of the forecasts to zero (forecasts collapse to a constant).[3] When such a point is reached, the mean squared out-of-sample forecast error collapses to the unconditional (historical) autocovariance of process Γ_j.

Cross-Hedging Fishmeal with Soybean Meal Futures

To test model performance, we derived a sequence of optimal intertemporal cross-commod-

[2] Notice the difference between the variance-covariance of forecast error (mean squared forecast error) and the variance-covariance of forecast. For example, the mean squared forecast error in (15) is $\boldsymbol{\Sigma}_{12}$, and the covariance between one-step and two-step-ahead forecasts is

$$\mathbf{CPA'C'} = \mathrm{cov}(\hat{\mathbf{y}}_{t+1}\hat{\mathbf{y}}'_{t+2}) = \mathrm{E}\{[\hat{\mathbf{y}}_{t+1} - \mathrm{E}(\hat{\mathbf{y}}_{t+1})][\hat{\mathbf{y}}_{t+2} - \mathrm{E}(\hat{\mathbf{y}}_{t+2})]'\} = \mathrm{E}(\mathbf{Cx}_{t+1}\mathbf{x}'_{t+1}\mathbf{A'C'}).$$

[3] Since the eigenvalues of the transition matrix $\mathbf{A}$ lie inside the unit circle (property of stationarity), by increasing k in (10), forecasts revert to the mean μ of the series.

ity hedging positions that could have been selected by a hypothetical fishmeal producer or dealer. Fishmeal is not traded in the futures market, so futures positions are established in soybean meal contracts instead. Lumpiness of futures contracts is ignored and brokerage commissions are subtracted *ex post*. Since the duration of fishmeal production may vary (typically from 1 to 4 weeks) depending on treatment of the freshly prepared meal, the fishmeal producer's hedging horizon may vary as well. To the extent that a change in hedging horizon may influence results, an empirical analysis is conducted with models of varying time periods. The three-period theoretical model developed above cannot be easily generalized to the *n*-period case, but it can be extended up to the five-period case without running into overly cumbersome calculations (Vukina 1991).

Midweek observations of menhaden fishmeal spot prices in Atlanta were collected from *Feedstuffs* and soybean meal data were collected from various issues of *The Wall Street Journal*. Futures data are Thursday closing prices of the Chicago Board of Trade soybean meal December contract. The December contract is the most actively traded soybean meal contract for which continuous price series exist throughout the year. The first trading day in December was used as the switching point between an expiring and incoming December contract. Data cover the five-year period between 5 June, 1986, to 23 May, 1991, for a total of 260 observations. The contemporaneous correlation coefficient estimate between the Atlanta fishmeal spot price and the CBOT December soybean meal futures is 0.8827, which is significantly different from zero at the 1% level. Such a correlation is relatively high, considering that for the same five-year period the correlation between Decatur soybean meal cash price and CBOT December futures price was only slightly higher (0.8931). This result justifies the use of the soybean meal futures for cross-hedging fishmeal cash positions.

In the first week, the dealer starts with fishmeal quantity q_0^* purchased at price p_0, which he is bound to deliver several weeks later. In case of a producer, p_0 can be interpreted as the accounting price (unit cost) of production, i.e., the price that would have been obtained had the product been ready for sale in period 0. Simultaneously, based on initial expectations, the initial futures position h_0^* is established. In the subsequent week, while still holding on to the initial inventory (or while production is still in process), the agent can change previous futures position h_0^* and execute a new position h_1^* based on updated information. In the final week, fishmeal is sold and the final futures position is liquidated. The model imposes no constraints on the magnitude and sign of the futures position. An optimal hedge could be speculative, meaning that both long futures positions and short futures positions greater than cash positions could be selected.

For the purposes of this example, cash position q_0^* is set at 100 tons of fishmeal (corresponding to one contract of CBOT soybean meal) and the weekly interest rate is assumed to be 0.15%, yielding approximately 8% per year.

Price Forecasting

The proposed framework leads to a multivariate time series forecasting model, with the Atlanta fishmeal cash prices and the CBOT December soybean meal futures prices modeled together. Parameters of the two-series state-space model are initially estimated using the first 104 observations (two years of data), and out-of-sample forecasts are generated using (10). Next, an additional observation (the 105th) is added to the existing data pool to update the system matrices, thus generating a new set of out-of-sample forecasts. The same procedure is repeated until available data are exhausted. In all rounds, the best results (smallest forecast errors) are obtained with autocovariance lag $j = 2$. A decaying pattern of singular values suggested the number of states n (rank of the Hankel matrix) is 2 in all instances.[4]

A summary of out-of-sample forecast statistics is presented in table 1. As one would expect, the best results are obtained with one-step-ahead forecasting. Comparison of the fishmeal cash price mean squared forecast error (244.5) with the corresponding zero-lag unconditional data autocovariance $\Gamma_0 = 4554$ indicates that the model reduces the out-of-sample total sum of squares by a factor of almost twenty. Larger forecast errors result when forecast distance increases. For example, the mean absolute percentage error (MAPE) of the one-step-ahead

[4] In state-space modeling of time-series, obtaining the stochastic process model from a given autocovariance model requires the solution to a special form of Riccati equation for the state covariance matrix $\mathbf{P}$. The Riccati equation has a positive definite solution if and only if the Fourier transform of the autocovariance sequence is positive semi-definite for all frequencies. If the condition is not met, we use the correction procedure from Vaccaro and Vukina.

Table 1. Out-of-Sample Forecast Statistics

Bivariate (cash and futures) Model With $j = 2$; $n = 2$; and updating system matrices	No. of observations: Initial	Final	No. of forecasts	Total variability Γ_0	Forecast statistics: MSE	MAPE
One-step-ahead forecasts	104	259	156			
Fishmeal cash price				4554.0	244.54	2.12%
Soybean meal futures price				945.3	90.23	2.59%
Two-step-ahead forecasts	104	258	155			
Fishmeal cash price				4554.6	465.10	2.98%
Soybean meal futures price				937.6	163.05	3.45%
Three-step-ahead forecasts	104	257	154			
Fishmeal cash price				4509.3	702.35	3.80%
Soybean meal futures price				909.1	242.39	4.11%
Four-step-ahead forecasts	104	256	153			
Fishmeal cash price				4395.2	996.24	4.36%
Soybean meal futures price				860.3	268.85	4.48%

soybean meal futures price forecast around its actual value is 2.59%; the MAPE of the two-step-ahead forecast is 3.45%; and so forth.

Cross-Hedging Ratios

Optimal cross-hedging ratios, defined as the ratios of the optimal soybean meal futures positions to the given fishmeal cash position, are reported in table 2. Our empirical analysis covers the last three years of the five-year period because the first two years of data were used to estimate the initial state-space model parameters. Results are the mean optimal cross-hedging ratios averaged over different numbers of sequences, with standard deviations reported in parentheses. Dynamic (multi-period) hedging ratios $D_0 = (h_0^*|Z_0)/q_0^*$; $D_1 = (h_1^*|Z_1)/q_0^*$; $D_2 = (h_2^*|Z_2)/q_0^*$; $D_3 = (h_3^*|Z_3)/q_0^*$ are compared with the static ratio ($S_0 = (h^*|Z_0)/q_0^*$).

Because the cash position is held constant (100 tons of fishmeal), a cross-hedge ratio of, say, 1.44 implies an optimal futures position of 144 tons of soybean meal. The static model yields hedging ratios that are, on average, speculatively short (greater than 1) for both 3-week and 5-week hedging horizons. The average optimal dynamic hedge ratios are also greater than 1 (except in the final time period of the hedging horizon), and they tend to decrease as the horizon approaches expiration.

Hedging decisions also depend on subjective risk parameter λ. For a risk-averse individual, λ can be any positive number. However, for sufficiently strong risk aversion, the second term in (5) may become inconsequential (expected returns on futures positions become unimpor-

Table 2. Optimal Cross-Hedging Ratios, 26 May, 1988, to 23 May, 1991

	Horizon of hedge							
	3 weeks–78 sequences			5 weeks–39 sequences				
Risk aversion	S_0	D_0	D_1	S_0	D_0	D_1	D_2	D_3
$\lambda \geq 0.1$								
Mean	1.08	1.44	0.52	1.38	2.35	1.60	1.44	0.51
(St.Dev.)	(0.16)	(0.15)	(0.24)	(0.12)	(0.26)	(0.14)	(0.16)	(0.23)
$\lambda = 0.01$								
Mean	1.09	1.46	0.54	1.39	2.36	1.62	1.45	0.53
(St.Dev.)	(0.17)	(0.16)	(0.27)	(0.12)	(0.23)	(0.16)	(0.17)	(0.27)
$\lambda = 0.001$								
Mean	1.21	1.60	0.73	1.51	2.48	1.81	1.58	0.73
(St.Dev.)	(0.35)	(0.43)	(0.74)	(0.29)	(0.36)	(0.42)	(0.37)	(0.91)
$\lambda = 0.0001$								
Mean	2.43	3.04	2.61	2.71	3.68	3.66	2.85	2.71
(St.Dev.)	(2.61)	(3.68)	(6.11)	(2.51)	(4.80)	(3.32)	(3.01)	(8.05)

tant), and the risk aversion parameter may not affect the hedging decision at all. Indeed, optimal hedges are found to be nearly identical for all values of λ greater than 0.1. Table 2 also summarizes optimal hedge ratios as risk aversion parameter λ decreases. Diminishing risk aversion results in higher average futures positions and greater instability. This result occurs because the relative weight placed on the expected returns from futures positions increases with decreasing risk aversion. If forecasts are correct, the larger the outstanding futures position, the bigger the profit. Therefore, speculating by hedging above the cash position, or even by going long, should become profitable.

Model Performance

Table 3 compares the returns (revenues and profits) generated by cash marketing and by various hedging schemes. Reported dollar figures are *ex post* evaluations of the terminal wealth function, with stochastic prices replaced by observed values. Under a cash marketing strategy, gains (or losses) result only from the difference between buying and selling cash price. Under routine hedging, a producer (dealer) always hedges 100% (takes a short position in the futures) of his cash position. In the static hedging scenario, only one hedge is initiated for the entire horizon. In the dynamic scenario, the futures position is adaptive and terminal wealth is determined by the optimal-hedge time path. Although brokerage commissions are not modeled, a scenario was also considered in which a \$50 commission per contract per roundturn is deducted *ex post* from gross returns. When evaluating the terminal wealth function, appropriate compound factors are used to account for the opportunity cost of capital. Total returns over all sequences, average returns per sequence, and standard deviations of returns are compared across two hedging horizons and various risk aversion parameters.[5]

Because of generally declining prices over the

Table 3. Comparison of Cross-Hedging Strategies, 26 May, 1988, to 23 May, 1991

Cross-hedging schemes	Revenues[a]						Profits[b]					
	3 week horizon			5 week horizon			3 week horizon			5 week horizon		
	Sum	Mean	(St.Dev)	Sum	Mean	(St.Dev)	Sum	Mean	(St.Dev)	Sum	Mean	(St.Dev)
Cash	−24,432	−313	(1,946)	−24,498	−628	(3,025)	−24,432	−313	(1,946)	−24,498	−628	(3,025)
Routine	−19,572	−251	(1,848)	−19,638	−504	(2,155)	−23,472	−301	(1,848)	−21,588	−554	(2,155)
Static												
$\lambda \geq 0.1$	−16,207	−208	(1,907)	−15,005	−385	(2,314)	−20,418	−262	(1,907)	−17,699	−454	(2,314)
0.01	−16,344	−210	(1,910)	−15,102	−387	(2,280)	−20,602	−264	(1,910)	−17,819	−457	(2,280)
0.001	−17,710	−227	(2,088)	−16,064	−412	(2,353)	−22,445	−288	(2,087)	−19,015	−488	(2,351)
0.0001	−31,379	−402	(8,436)	−25,689	−659	(14,167)	−40,873	−524	(8,480)	−30,980	−794	(14,193)
Dynamic												
$\lambda \geq 0.1$	−15,520	−199	(1,983)	−15,435	−396	(2,543)	−23,176	−297	(1,983)	−26,978	−692	(2,543)
0.01	−14,722	−189	(1,949)	−14,730	−378	(2,500)	−22,508	−289	(1,949)	−26,397	−677	(2,499)
0.001	−6,740	−86	(2,407)	−7,682	−197	(3,708)	−15,823	−203	(2,395)	−20,580	−528	(3,677)
0.0001	73,083	937	(18,907)	62,805	1,610	(32,498)	51,029	654	(18,700)	37,586	964	(32,170)

[a] Revenues (in \$) are based on $q_0^* = 100$ tons of fishmeal.
[b] Profits (in \$) are calculated as revenues minus \$50.00 brokerage commission per contract per roundturn.

[5] Differences across models in average returns within the same hedging horizon were tested using a paired observations procedure (Walpole and Myers, pp. 252–55). These results were not highly significant, with t-statistics generally between $t_{0.6}$ and $t_{0.9}$ percentiles. The increase in the static and dynamic models' variablity of returns over the routine model's variability of returns was generally significant at the 1% level ($\chi^2_{0.99}$) for small values of λ(0.001 and 0.0001), but not significant for $\lambda = 0.1$ and $\lambda = 0.01$.

period analyzed, the cash marketing strategy generated negative average returns with fairly large standard deviations. Routine hedging reduced average losses and their standard deviations relative to the cash marketing strategy. Hedging the entire inventory over the planning period appeared to be a reasonable method of risk reduction. A similar result has been seen elsewhere in the literature (Peck). A routine hedge also compares favorably with static and dynamic hedges in terms of limiting revenue variability. When mean revenues are compared, however, routine hedging seems to be generally inferior to both forecasting-based strategies.

There is a tendency for the static hedging model's performance to deteriorate as λ decreases. Because they ignore the dynamic character of a decision process, static hedge ratios are in fact suboptimal. With decreasing risk aversion, more weight is placed on the futures positions' expected returns, so that the static model's total performance worsens. Because of cash market losses, static model returns stayed negative for all values of λ. Still, for larger λ values (0.1 to 0.001), static hedging resulted in smaller losses and lower variability than did cash marketing, and in smaller losses but higher variability then did routine hedging. For very small λ, a static hedge appears to be inferior to both cash marketing and routine hedging. However, because of the large standard deviation of the static model's returns, differences in average losses among models for $\lambda = 0.0001$ are statistically insignificant.

In contrast to the static model, mean returns from dynamic hedging improve as λ decreases. For example, with $\lambda = 0.1$, the difference in mean revenues between the dynamic and static models is not statistically significant, while with $\lambda = 0.0001$ the dynamic strategy yields positive returns. For both dynamic and static models, variability of returns increases with decreasing λ, indicating that the less risk-averse individual will accept higher risk in order to achieve higher expected profit.

Comparison of results for the two different hedging horizons suggests that the performance of the static hedge worsens as the horizon gets longer. This occurs because the optimal static futures position does not change as new information become available, so that the approximation of a truly time varying hedge by a single static hedge deteriorates with the extension of time horizon.

Ex post deduction of brokerage commissions from revenue figures did not change variability of returns in any of the models. However, the overall dynamic model's performance became inferior to that of the static model, except in the case of a minimally risk-averse trader. For $\lambda = 0.0001$, profits generated by dynamic hedging were positive even after accounting for average commission fees of $646.

Conclusions

We hypothesized that, within acceptable risk levels, economic agents may increase average marketing returns by combining information from price forecasting models with an appropriate hedging strategy. Lower risk aversion implies increased weight is placed on the expected returns from holding futures positions. Hence, if forecasts are accurate, less risk-averse traders can potentially earn larger profits. Because optimal hedge ratios are derived as a trade-off between expected value and the variance of returns, increased average profitability is accompanied by increased variability.

In the static model, our findings are consistent with previous research (Peck). As risk aversion decreases, average returns decrease and variability rises, suggesting that forecast-based static strategies are generally unprofitable. This result is explained primarily by the inadequacy of the static model to simulate an intrinsically dynamic decision process such as hedging. In the dynamic model, however, lower risk aversion results not only in higher variance of returns, but also in higher average returns. In the latter case, price forecasts generated by the state-space procedure are accurate enough to increase, within acceptable risk levels, the average returns from hedging and/or speculating.

The forecasting-based dynamic hedging model presented here appears to improve the ability of a less risk-averse fishmeal producer (dealer) to increase marketing returns. Two model extensions might be interesting. In our simplified approach, brokerage commissions were subtracted *ex post*, significantly affecting the dynamic model's performance. Since commissions may influence the decision process itself, they should be incorporated into the optimal solution. And, because the variance of distant out-of-sample forecasts of a stationary time series eventually collapses to the historical variance of the random process, a point may be reached after which length of hedging horizon becomes inconsequential. This creates a possibility of generalizing the model to an n-period case.

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