Direct asymptotic equivalence of nonparametric regression and the
infinite dimensional location problem

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Abstract

We begin with a random design nonparametric regression having random predictors and Gaussian errors. We produce a convenient, easily implementable mapping of this problem into a Gaussian infinite dimensional location problem. Such an infinite dimensional problem can reflect a Fourier, or wavelet, or other orthogonal basis representation of the original regression situation. In this way it may be easier to analyze than the original regression formulation. There is considerable literature on doing this; beyond describing the situation we do not pursue here this issue of the analysis of such infinite dimensional models. For most of our results the random regressors in our theory may have either a known or unknown distribution.

The correspondence we produce between the regression and location problems is an asymptotic equivalence mapping. (We also explicitly describe the converse mapping from the location problem to the regression.) Thus any solution to a statistical problem in one formulation can be easily converted to a solution for the other formulation.

The basic mapping from the regression to location formulations involves a few steps. First, bin the regression observations and use the bin averages to compute an empirical infinite series transform. Then truncate this series appropriately. Add a small amount of prescribed Gaussian noise to the truncated series coefficients. Then use a subset of these to linearly predict the remaining tail coordinates of the infinite series. In many applications the latter two steps are not necessary even though they are needed for an explicit asymptotic equivalence mapping.

Key words: Nonparametric regression; Random design; Fourier series; Wavelet; Equivalence;

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