Statistics 430  
First Midterm, Spring 2002

Write your answers in the supplied blue books. You must show all work related to the solution in order to receive full credit. The items (not the questions) are approximately equally weighted.

1. In the dice game craps, a player rolls two six-sided dice and adds the two outcomes to determine a score for that roll. Here are the rules:

   • On the first roll, you win if you roll a 7 or 11. You lose on the first roll if you roll a 2, 3, or 12.

   • If you have not won/lost on the first roll, then your score on the first roll becomes your “point” for later rolls. You win on subsequent rolls if you roll your “point” without having rolled a 7 and lose if you roll a 7 without rolling your “point”.

Assume that the rolls are independent.

(a) Describe the sample space for the first roll of the two dice.

(b) What is the probability that you win or lose on your first roll?

(c) What is the probability that you win if your first roll yields a “point” of 8.

2. Two roommates share an apartment. They keep their batteries in a drawer in the kitchen, not in their two flashlights. They have 10 batteries, but 4 of these have gone bad. When the power goes out, they randomly select two batteries and put them in the first flashlight. Then they pick two more batteries at random and put them in the second flashlight. A flashlight works only if both batteries are good.

(a) What is the probability that both flashlights work?

(b) What is the probability that the second flashlight works?

(c) What is the probability that the second flashlight works if the first works?

3. In basketball, smaller players often attempt to “fake” a larger defender to get a clear shot at the basket. When being defended by a smaller player, however, larger players seldom need such a ruse. Suppose that when faced with a larger defender in such a matchup, smaller
players use a fake 75% of the time whereas larger players never fake when being guarded by smaller players. When players of similar size meet up, it’s anyone’s guess what might happen – fakes occur 50% of the time.

For this league, assume that 60% of the matchups are between players of similar size, with the rest evenly divided between matchups between larger offensive and smaller defensive players and between smaller offensive and larger defensive players.

(a) Jane plays in this league and just took a shot. What is the probability she used a fake?

(b) Anne just used a fake to get off a shot. What is the probability that she was facing a larger player?

4. Electrons can either spin “up” or “down”. Suppose that we measure the spin using one of three directions. Let $U_1$ denote the event that the spin was “up” when measured from the first direction, $U_2$ denote the event that the spin was “up” when measured in the second direction, and similarly $U_3$ denote the event that the spin was “up” when measured from the third direction. It is known that

$$P(U_1) = P(U_2) = P(U_3) = \frac{1}{2}.$$ 

Other experiments have shown that

$$r = P(U_1 \cap U_2) = P(U_1 \cap U_3) = P(U_2 \cap U_3).$$

Let $s = P(U_1 \cap U_2 \cap U_3)$.

(a) Find $P(U_1 \cup U_2 \cup U_3)$ in terms of $r$ and $s$.

(b) Using the facts that $P(U_1 \cup U_2 \cup U_3) \leq 1$ and $s \geq 0$, find an inequality for the size of $r$.

(These are Bell’s inequalities from physics, and it’s an uncomfortable fact that $r$ is measured to be about 0.1.)

5. There are three chests, each having two drawers. Each drawer contains either a gold coin or a silver coin. Chest 1 has two gold coins, Chest 2 has a gold coin and a silver coin, and Chest 3 has two silver coins. A chest is picked at random and one of its drawers opened (again, at random). If one observes a silver coin in the drawer, what is the probability that the other drawer in this chest has a silver coin?